Introductory Course to Matlab with Financial Case Studies

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Lefkosia, September 2003
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1. Introduction
This handout demonstrates a comprehensive introduction to the basic utilities of a high technical programming language encapsulated under the computing environment of Matlab. It was prepared based on version 6.5 Release 13, but since it does not have many differences from previous ones, it can also operate for older versions as well. Upon reading it, you will be in a position to create programming code to solve elementary (or even intermediate) financial problems. It has been prepared for postgraduate students registered for the Master in Finance program offered by the Department of Business and Public administration – University of Cyprus.

1.1 Learning Matlab
Matlab (the name stands for: Matrix Laboratory) is a high performance programming language and a computing environment that uses vectors and matrices as one of its basic data types (MATLAB® is a registered trademark of the MathWorks, Inc.). It is a powerful tool for mathematical and technical calculations and it can also be used for creating various types of plots. It performs the basic functions of a programmable calculator whereas someone can write, run/execute and save a bundle of commands. In the last few years, Matlab has become very popular because it enables numerous ways to solve various problems numerically via a user-friendly programming language. It is particularly easy to generate a programming code, execute it to get the desire solution, draw some relevant graphs and look at the interesting features of the problem under consideration. What makes Matlab so convenient to many fields (including finance) is that it integrates computation, visualization and programming in an easy to use environment. Just for historical purposes, the first version of Matlab was written in 1970s by a numerical analyst names Cleve Moler, and since then has become a successful retail product.
Matlab features a family of add-on application-specific solutions called toolboxes. A toolbox is a comprehensive collection of Matlab functions (M-files) that extend the Matlab environment to solve particular classes of problems. For example, the Financial Toolbox includes ready to use functions that provide a complete integrated computing environment for financial analysis and engineering. The toolbox has everything you need to perform mathematical and statistical analysis of financial data and display the results with presentation-quality graphics. With MATLAB and the Financial Toolbox, you can: compute and analyze prices, yields, and sensitivities for derivatives and other securities, and for portfolios of securities; analyze or manage portfolios; design and evaluate hedging strategies and many more.

This handout is about a brief introductory course in the most important parts of Matlab. After finishing this, you will be able to:

i. Utilize the basic mathematical operations;
ii. Get know the general purpose commands of Matlab;
iii. Become familiar with some of the elementary functions, matrix and numerical linear algebra functions, as well as some graphic and plot commands;
iv. Manipulate matrices (i.e. create and edit vectors and matrices, build a larger matrix from a smaller one, etc);
v. Learn how to use the relational operators (<, >, <=, >=, ==, ~=) and logical operators (& AND, | OR, ~ NOT);
vii. Recognize built in variables, define new ones and performing computations with them;
vii. Learn how to use \texttt{if} and \texttt{switch} statements;
viii. Learn how to correctly utilize loop commands, such as the \texttt{for} loop, and the \texttt{while} loop;
ix. Write and edit your own m-files and functions for solving a specific problem;
x. ...Numerous other utilities and uses depending on your will to learn and utilize this technical language.
It is better to use this handout in direct use with the Matlab. It will be more beneficial if while reading these notes you sit in front of a PC and type the various commands and execute the given examples.

Although the Matlab package includes extensive help facilities that can be viewed via a very user-friendly Help Browser, if it is needed, you can also find additional online (through Internet) help at the following electronic address:


or at:

http://wwwhost.cc.utexas.edu/math/Matlab/Manual/ReferenceTOC.html

Moreover, various download, trials and the Matlab Central File Exchange that contains hundreds of files contributed by users and developers of Matlab and related products can be found at:

http://www.mathworks.com/web_downloads/

Additionally, any functions that have been created for examples and all those that are needed to work with the case studies are located in the folder named as: Matlab Examples (for further details send me an email or ask during the class).

Lastly, I will appreciate to send me feedback information concerning these notes related with spelling errors, sections that should be clarified or explained better (or even added) and any other that can contribute to the improvement of this handout. The email address that you can reach me is:

benjamin@avacom.net.
1.2 Basic Definitions

Before moving to the introduction of issues specific to Matlab, some very basic definitions on computers and programming should be set forth. These include:

General [1]

- **bit** (short for binary digit) is the smallest unit of information on a computer. A single bit can hold only one of two values: 0 or 1. More meaningful information is obtained by combining consecutive bits into larger units, such as byte.
- **byte** consists a unit of 8 bits and is capable to hold a single character. Large amounts of memory are indicated in terms of kilobytes (1024 bytes), megabytes (1024 kilobytes), and gigabytes (1024 megabytes).
- **data** is information represented with symbols such as numbers, words, images, etc.
- **command** is a collection of programming words that instruct the computer to do a specific task.
- **algorithm** is a set of commands that aim to solve a predetermined problem.
- **program** is the implementation of the algorithm suitable for execution by a computer.
- **variable** is a container that can hold a value. For example, in the expression: \( z + a \), both \( z \) and \( a \) are variables. Variables can hold both numerical data (e.g. 10, 98.5) and characters or strings (e.g. 'd' or 'pba').
- **constant** is a value that never changes. It can be a numeric, a character or a string.
- **bug** is an error in a program, causing the program to stop running, not to run at all or to provide wrong results. Some bugs can be very subtle and hard to find. The process of finding and removing the bugs is called **debugging**.

Matlab Specific [2]

- **workspace** is the memory allocated to Matlab and is used to temporarily store variables.
- **m-file** is a file that contains Matlab’s language code. *m-files* can be **functions** that accept arguments and produce output, or they can be **scripts** that execute a series of Matlab statements. For Matlab to recognize a file as an *m-file*, its file name extension must be *.m.*

- **functions** are *m-files* that can accept input arguments and return output arguments. The name of the *m-file* and the calling syntax name of the function should be the same. Functions operate on variables within their own *workspace*, separate from the *workspace* you access at the Matlab **command prompt**. Functions are useful for extending the existing Matlab language for personal applications (e.g. create a function that returns the expected return and standard deviation related with a set of companies).

- **scripts** can operate on existing data in the *workspace*, or they can create new data on which to operate. Although scripts do not return output arguments, any variables that they create remain in the *workspace*, to be used in subsequent computations. In addition, scripts can produce graphical output using **functions**. Scripts are useful for automating a series of steps that are needed to be performed many times (e.g. to create a script that executes a series of functions related to portfolio optimization).

- Every **variable** has a **name** and a **data type**. With Matlab, accepted **variables names** do not start with symbols (like ~, +, -) or numbers, use lower and upper case letters do not exceed 63 characters and do not resemble **reserved words** and **build-in functions**. Acceptable definitions include: *Time, x, y, XYZ, Ray_value, U3e23* etc. Non-acceptable definitions are the followings: +Time, 3587num, _XYZ, rayX-values, for, end, while, case, day etc. The **data type** is a classification of particular type information. Among the most usable types are: **integer** a whole number, a number without any fraction (e.g. 12, 10, 89); **floating point** a number with a fractional part (e.g. 25.7, 78.1, 0.000005, 5e-5 which is equal to 5*10^-5); and **character** readable text character (e.g. ’k’). With Matlab, it is not need to type or declare variables used in the *m-files*. Any operation that assigns a value to a variable creates the variable, if needed, or overwrites its current value, if it already exists.
• Every build in or user made function has a calling syntax that is unique for each function. In Matlab, to call a function you type the function’s name and you enclose the input arguments in brackets (if more than one input argument exist, then commas should be used to separate the expressions).

• Matlab is handled through the use of various windows, that each is related with a certain utility. For example, the Command Window is used to enter variables and run functions and m-files, the Command History Window makes a diary with the commands that you have recently entered in the command window, the Workspace Browser allows you to view the variables that are stored in the workspace, etc. Any reference to such windows that is made in the main body of these notes will be explained in detail if it is needed.

1.3 Information of How to Read This Handout
In the main body of this handout, words that refer to a keyboard instruction will be written in brackets and in boldface letter. For example, [Enter] will represent one key-press of the Enter button of the keyboard. Words that represent Matlab’s reserved words such as for, end, who, etc, words that imply a Matlab term such as m-file, script, function, etc, or key-words related with the interface menu and toolbars will appear in italics. Words in the main body that refer to Matlab’s vector and matrix names, Matlab’s build in functions or user’s new functions will be presented with a shadow style. For example, if in the main body a reference is made to a vector saved in the workspace with the name “Hours” it will look like: Hours. The reference to Matlab’s function that creates 2D figures will be as: plot. Moreover, note that although each function has a specific calling syntax, usually only the name of the function will be displayed; if not presented in the body text of this handout, the user should be responsible to find the exact calling syntax of the function via the Matlab’s help facilities.

Other Matlab output such as warnings, tips or Matlab commands will be enclosed in double quote marks “ ”. The definition of Matlab related words and expression in windows (as you will see later) consist exceptions from these rules. Lastly, references to books and other reading material are
numbered in square brackets (e.g. [2] is the Matlab online help facilities; see the section with the references). Additionally, if a figure or a table heading that has a superscript number enclosed in square brackets will indicate the source from where it was copied (for instance, Figure[2] X: Figure heading, indicates that the current figure was copied from Matlab online help facilities).

1.4 Starting Up Matlab

The following instructions help for starting up the Matlab in the Economics and Business Computer Lab:

**Step #1:** Use the [Ctrl]+[Alt]+[Del] combination to bring up the logon screen (at this point you should enter the user name and your password and after to press [Enter])

**Step #2:** After few seconds, you view the PC’s Desktop screen with all available icons. Find the Matlab’s shortcut icon (labeled as “Matlab” and looks like 📖) and double click on it. After few moments, the Matlab starts up and the following sentence appear in one of the screens:

“To get started, select "MATLAB Help" from the Help menu”

**Step #3:** The Matlab is now ready to be used (if you want to quit Matlab, from the window named as Command Window either type quit or exit from the toolbar choose: File > Exit Matlab).

1.5 Matlab’s Windows

When you start up Matlab, you will view the following interface (Figure 1 – a difference interface may appear if someone before you has changed the active windows from the View menu):
The above is termed as the Matlab Desktop (graphical user interfaces) and contains tools for managing files, variables, and applications associated with Matlab. Think of the desktop as your instrument panel for Matlab. The toolbar in the Matlab desktop provides easy access to frequently used operations. Hold the cursor over a button and a tooltip appears describing the item. Note that some of the tools also have toolbars within their windows.

You can change the way your desktop looks by opening, closing, moving, and resizing the tools in it. Use the View menu to open or close the tools. You can also move tools outside the desktop or move them back into the desktop (docking). All the desktop tools provide common features such as context menus and keyboard shortcuts. You can specify certain characteristics for the desktop tools by selecting Preferences from the File menu. For example, you can specify the font characteristics for command
window text. For more information, click the Help button in the Preferences dialog box [2].

Additionally, two more windows that can be seen in Figure 1 can help the user during the programming time. The first one and most important is the workspace browser (Figure 2). The Matlab workspace consists of the set of variables (named arrays) built up during a Matlab session and stored in memory. You add variables to the workspace by using functions, running m-files, and loading saved workspaces.

![Figure 2: The Matlab Workspace Browser](image)

The workspace is not maintained after you end the Matlab session. To save the workspace to a file that can be read during a later Matlab session, select Save Workspace As from the File menu.

Another useful window, is the command history (Figure 3). Statements you enter in the Command Window are logged in the Command History. In the Command History, you can view previously run statements, and copy and execute selected statements.
To activate or close these and some other windows, choose View from the toolbar. In the following few subsections, the most basic from this screen are documented.

1.5.1 Command Window – Matlab as a Calculator

It is the main window in which the user communicates with the software. In the command window, the user can view the prompt symbol “>>” which indicates that Matlab is ready to accept various commands by the user. Via this window, the user can employ the basic arithmetic operators like: “+” (addition), “-” (subtraction), “*” (multiplication), “/” (division), “^” (powers) and the “)” (brackets), as well as many other build in elementary and other functions and commands that will be referred to later.

As a first example, enter the following command that performs a basic calculation (Figure 4):

“5+6/12+9/3-21”
Figure 4: A basic calculation

The Matlab displays the results into a variable named as ans. This is a default variable name that is used by the command window to display the most recent results instructed by the user that has not defined a specific name. Continue by entering the following command that creates a four-element vector (Figure 5):

"[1 2 3 4]"

Figure 5: A four-element vector
The square brackets "[ ]" indicate the definition of the vector. Also, the space between the vector numbers separates the vector's elements. The comma "," is another way of separating the elements. Additionally, note that the default variable ans has lost its previous value in order to store the results of the most recent operation.

As another example, enter the following command that creates a 3-by-3 magic square saved in the matrix variable M:

"M=magic(3)"

and after pressing [Enter] you get the Matlab’s result (Figure 6):

![Command Window](image)

Figure 6: A 3-by-3 magic table

The magic square is created using the element function magic that is already built in Matlab.
Type also the following:

```
x=[(2^2+1)^2-15/4*6.1, 1.23e-2]
```

and after pressing [Enter] you get the Matlab's result (Figure 7)

![Command Window](image)

**Figure 7:** A 2-element vector

Observe that the very first command that you have entered in the command window has vanished with the additional of the last one (of course it depends on the size of the window, in here the command window is minimized so earlier commands vanish too quickly). You can use the scroll bar on the right to navigate the command window and go up to see the earlier commands (or view them via the command history window).

Matlab calculates the quantities as follows:

- First come the quantities in brackets
- Power calculation follow: (e.g. $5 + 2^2 = 5 + 4 = 9$)
• “*” and “/” follow by working from left to right: (e.g. 2*8/4 = 16/4 = 4)
• “+” and “-” follow last, from left to right: (e.g. 6-7+2= -1+2 = 1)

Note that “e” notation is used for very large or very small numbers. By definition: 1e1=1X10^1 and 1e-1=1X10^-1.

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<thead>
<tr>
<th><strong>Trigonometric</strong></th>
<th><strong>Exponential</strong></th>
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<tr>
<td>sin</td>
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<td>sinh</td>
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<td>asinh</td>
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<td>rem</td>
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<td>sign</td>
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</table>

**Table 1:** The elementary build-in functions

Matlab can handle three different kinds of numbers: integers, real numbers and complex numbers (with imaginary parts). Moreover, it can handle non-number expressions like: NaN (Not-a-Number) produced from mathematically undefined operations like: 0/0, ∞*∞ and inf produced by...
operations like 1/0. Matlab as a calculator includes a variety of build-in mathematical functions like: trigonometric, exponential, complex, rounding and remainder, etc. Table 1 below, depicts these elementary mathematical build-in functions (to learn how to use these commands, review the Matlab help facilities located in section 1.5.3).

Note the build-in functions exhibited in Table 1 have their own calling syntax. For example, if you type sin in the command window, Matlab returns the following error massage:

“??? Error using ==> sin”
“Incorrect number of inputs.”

This happened because the sin function requires an input expression like a number enclosed in brackets. If you enter:

“sin(5)”

then you get an answer:

“ans =
   -0.9589”

In the rest of this handout, only the name of the build in functions will be given. Beside some exceptions where the correct calling syntax of a function is fully tabulated, the user is responsible in knowing the necessary input arguments to the function. Additional the use of the command window will be apparent as you read this manuscript since the learning of a computer programming language is pure a “learning by doing” process.

1.5.1.1 Controlling Command Window Input and Output
This section presents ways by which the user can control the command window input and output.
List of Useful Commands:

- **who**: Lists current variables located in the *workspace*.
- **whos**: A long form of who. It lists all the variables in the current *workspace*, together with information about their size, bytes, class, etc.
- **clear**: Clear variables and *functions* from memory.
- **home**: Moves the cursor to the upper left corner of the *command window* and clears the visible portion of the window. Use the scroll bar to see what was on the screen previously.
- **clc**: Clears the *command window* and homes the cursor.
- **quit**: Terminates Matlab.

The format function:
The *format* function controls the numeric format of the values displayed by Matlab. The *function* affects only how numbers are displayed, not how Matlab computes or save them. Here are the different *formats*, together with the resulting output produced from a vector *x* with components of different magnitudes [2]. In the command window type:

```
format short
```

and afterwards:

```
x
```

for retrieving the *x* vector that you have created before to get:

```
x =
  2.1250   0.0123
```

Note that with this format only 4-decimals place are being used (in total 5 digits). Try also the following format commands to see what you get:

- **format short e**: Floating-point format with 5 digits.
- **format short g**: Best of fixed or floating-point format with 5 digits.
format long e
Floating-point format with 15 digits.

format long g
Best of fixed or floating-point format with 15 digits.

format bank
Fixed format for dollars and cents.

**Suppressing Output**
If you simply type a statement and press [Enter], Matlab automatically displays the results on screen. However, if you end the line with a semicolon “;” Matlab performs the computation but does not display any output [2]. This is particularly useful when you generate large matrices. For example,

“A = magic(100);”

although Matlab creates a 100-by-100 matrix saved in the *workspace*, the result is not displayed on the *command window*. Moreover, when a command ends with “;”, is allowed to enter additional commands before pressing the [Enter] key. For instance,

“A = magic(100); B=A;”

will perform two commands the one after the other. In here, after creating the 100-by-100 magic square that is stored in the A (matrix) variable, matrix B is created to hold the same values as with A. Additionally, note that if two or more statements are separated with commas, Matlab will display the results in the screen in the order that these statements were entered.

**Entering Long Statements**
If a statement does not fit on one line, use an ellipsis (three periods), “...”, followed by [Enter] to indicate that the statement continues on the next line. For example,

“s = 1 -1/2 + 1/3 -1/4 + 1/5 - 1/6 + 1/7 ...
    - 1/8 + 1/9 - 1/10 + 1/11 - 1/12”

to get:
“s =
0.65321”
Blank spaces around the “=” and “+”, and “-“ operators are optional, but they improve readability.

**Command Line Editing**
Various arrow and control keys on your keyboard allow you to recall, edit, and reuse statements you have typed earlier. For example, suppose you mistakenly enter:

“rho = (1 + sqrt(5))/2”

You have misspelled square root: sqrt. Matlab responds with:

“Undefined function or variable ‘sqrt’”

Instead of retyping the entire line, simply press the ↑ key. The statement you typed is redisplayed. Use the ← key to move the cursor over and insert the missing “r”. Repeated use of the ↑ key recalls earlier lines. Typing a few characters and then the ↑ key finds a previous line that begins with those characters. You can also copy previously executed statements from the command history window [2].

**Interrupting a Command, a Script or a Function**
It is often that a user writes a command (or script or a function) that enters to an endless loop, so it keeps running. Pressing [Ctrl]+[C] once or few times, it will terminate the execution. Sometimes where the user runs a script that calls other functions, the interruption with [Ctrl]+[C] might take few seconds to few minutes to take place.

**Current Directory**
On the command window toolbar, the user can find the current directory address bar. Matlab file operations use the current directory and the search path as reference points. Any file you want to run must either be in the
current directory or on the search path (the search path is a default list of paths that include all folders with Matlab build in functions and toolboxes; to access this window from the command window go: File>Set Path.). A quick way to view or change the current directory is by using the current directory field in the desktop toolbar as shown in Figure 8. To search for, view, open, and make changes to Matlab-related directories and files, use the Matlab current directory browser which is called after clicking the icon: ....

![Figure 8: Command Window Toolbar with Current Directory](image)

1.5.2 Editor/Debugger

Use the Editor/Debugger to create and debug m-files, which are programs you write to run Matlab functions [2]. The Editor/Debugger provides a graphical user interface for basic text editing, as well as for m-file debugging (Figure 9).

![Figure 9: Matlab’s Editor/Debugger](image)
It’s like a text editor (e.g. MS Word) suitable for writing executable Matlab code. Notice that reserved words (e.g. while, if, else, end, function etc) appear in blue styling whilst accompanied comments in the programming code (that can be placed after the comments symbol: “%”) appear in green letters [2]. Exploit the various menus to become familiar with this window. For example, by selecting: File> New you can create a new m-file or by selecting: Debug> Run you can run a saved script that includes various commands and functions.

1.4.3 Help Options
Matlab is technical software that is enhanced with extensive online help via various help facilities.

help Command
In first place, if the user knows the topic in which an informative help tip is needed it can use the help command. For instance, if help is needed about the magic squares, the user types the following expression in the command window:

“help magic”

and after pressing the [Enter] key, the Matlab response is (Figure 10):

![Command Window](image)

**Figure 10:** Matlab’s help response on magic squares
Type also:

“help elfun”

to view the help results concerning the build-in elementary functions exhibited in Table 1 (notice also that elfun is a reserved word).

The problem with the help command is that the user must be familiar with the topic under consideration and the word following the help command must be exact and spelled correctly. For example, the Matlab function that returns the inverse of a matrix is named as: inv (type “help inv” to see what you will get). If the user types “inverse” or “inverce” instead, then Matlab help will return that such functions cannot be found.

lookfor Command
A more flexible command for perusing help from Matlab, is the lookfor. If the user is not familiar with the exact help that is looking, it can use the lookfor, which looks for the given string in the first comment line of the help text in all m-files located in Matlab’s toolboxes. For example, type:

“lookfor inverse”

and Matlab returns all m-files functions that contain in their introductory comments the word “inverse”. Among them, the user can locate the correct name for the inverse function (which is simply: inv) and with the help command can see in detail what this command does. Note that the lookfor command is time consuming and sometimes takes up to some minutes to come up with a result.

The Help Window/Browser
Online help (online here does not refer to any Internet source) can also be obtained via the Help menu found in the Matlab’s desktop. From the desktop window select Help>Matlab Help to get the help browser (Figure 11) with a list of help topics. Through this screen, the user can navigate around a variety of topics by double clicking on them (this browser displays html help pages and can be operate like the Internet Explorer).
To become familiar with the Matlab's accompanied toolboxes, navigate in the contexts, open up the dropping down choices and get know various topics.

Figure 11: Matlab's help browser
2. Manipulating Vectors and Matrices

A matrix or an array is the basic element on which Matlab can operate. A 1-by-1 matrix forms a scalar or a single number, whereas a matrix with only one row or column forms a row or column vector respectively. This section exhibits the mathematical manipulation of vectors (arrays) and of two-dimensional matrices.

2.1 Row Vectors

A vector is a list of numbers separated by either space or commas. Each different number/entry located in the vector is termed as either element or component. The number of the vector elements/components determines the length of the vector. In Matlab, square brackets “[]” are used to both define a vector and a matrix. For instance, the following command returns a row vector with 5 elements:

Matlab’s command:

>> y=[ 5 exp(2) sign(-5) sqrt(9) pi]

Matlab’s response:

y =

5  7.3891   -1   3  3.1416

Note that the definition of the row vector, the user it free to use any built-in function as long as this is used properly. In the above definition, exp is the exponential, sign returns the sign, sqrt is the square root and pi represents p. General speaking and except some special cases, when a function is applied to a 1-by-1 scalar, the result is a scalar, when applied to a row or column vector is a row or column vector and when applied to a matrix the output is again a matrix. This happens because Matlab applied the build-in
functions element-wise. The length of the above vector is obtained via the following command:

**Matlab's command:**

```matlab
>> length(y)
```

**Matlab's response:**

```
ans =
5
```

Note that length is a build-in function that given a vector, it returns its length. Since the result of this function is not stored in a user-defined variable, Matlab saves the result in the ans variable. If it is need to save the vector’s length to a variable named Ylength the command would be:

**Matlab's command:**

```matlab
>> Ylength=length(y)
```

**Matlab's response:**

```
Ylength =
5
```

With Matlab, vectors can be easily multiplied by a scalar and added or subtracted with other vectors of similar length. Moreover, a scalar can be added or subtracted to or from a vector and smaller vectors can be used to construct larger ones. All these operations are performed element-wise. Note that each vector represents a variable. Some examples follow:

**Matlab's command:**

```matlab
>> a=[-1 2 -3]; b=[2 1 2];
>> c=5*a
```

**Matlab's response:**

```
c =
   -5    10   -15
```

**Comments:**

An element-by-element multiplication of vector a by 5.
Matlab's command:
>> c1=5+b

Matlab's response:
c1 =
  7  6  7
Comments:
An element-by-element addition of vector b with 5.

Matlab's command:
>> d=a+b

Matlab's response:
d =
  1  3 -1
Comments:
An element-by-element addition of two equal length vectors.

Matlab's command:
>> e=[2*a, c, d]

Matlab's response:
e =
-2  4 -6 -5  10 -15  1  3 -1
Comments:
A larger vector is created after certain manipulations.

To refer to specific elements of the vector, the vector's name is followed by the element’s rank in brackets. For instance we can change the values of the e vector with the following command:

Matlab's command:
>> e(2)=-99; e(4)=-99; e(6)=-99;
>> e

Matlab's response:
e =
-2 -99 -6 -99 10 -99 1 3 -1

2.1.1 The Colon Notation
The colon notation “:” can be used to pick out selected rows, columns and elements of vectors, matrices, and arrays. It is a shortcut that is used to create row vectors. Moreover, the colon notation is used to view or extract
parts of vectors (afterwards, with matrices, the colon can be used to view a certain part of a matrix). For instance, the following commands produce three different row vectors:

**Matlab's command:**

\[
>> v1=1:8, v2=-4:2:2, v3=[0.1:0.2:0.6]
\]

**Matlab's response:**

\[
\begin{align*}
v1 &= 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
v2 &= -4 & -2 & 0 & 2 \\
v3 &= 0.1 & 0.3 & 0.5
\end{align*}
\]

The careful viewer should have notice that the vector creation via the use of the colon notation has the form: *starting value*: *step*: *finishing value*. The *starting value* consists the first value/element of the vector, the *finishing value* is the last value/element of the vector and all other elements differ by a value equal to *step*. Also, when the interval between *finishing value* and *starting value* is not divisible by the *step*, the last value of the vector is not the *finishing value* (i.e. in v3 the last element is 0.5 and not 0.6). The colon notation is also used to view or extract parts of a vector. See the examples to get an idea.

**Matlab's command:**

\[
>> v4=v1(2:4)
\]

**Matlab's response:**

\[
\begin{align*}
V4 &= 2 & 3 & 4 
\end{align*}
\]

**Comments:**

Extracting the 2\textsuperscript{th}, 3\textsuperscript{rd} and 4\textsuperscript{th} elements of v1.

**Matlab's command:**

\[
>> v1(2:3:8)
\]

**Matlab's response:**

\[
\begin{align*}
\text{ans} &= 2 & 5 & 8 
\end{align*}
\]

**Comments:**

Extracting the 2\textsuperscript{nd}, 5\textsuperscript{th} and 8\textsuperscript{th} elements of v1.
2.1.2 Column Vectors

Column vectors are created via the use of semi-colon ";" instead of commas and spaces. Operations with column vectors are similar as with row vectors. The following examples depict the manipulation of column vectors.

**Matlab's command:**

```matlab
>> cv=[1;4;7;9]
```

**Matlab's response:**

```
cv =
 1
 4
 7
 9
```

**Comments:**

Creating a four-element column vector.

**Matlab's command:**

```matlab
>> length(cv)
```

**Matlab's response:**

```
ans =
 4
```

**Comments:**

Taking the vector's length.

**Matlab's command:**

```matlab
>> CV=[cv(1:2)+2; cv(2:3)*5]
```

**Matlab's response:**

```
CV =
 3
 6
20
35
```

**Comments:**

Creating CV that is a four-element column vector. Its first two elements are the two first elements of cv increased by 2 whereas the last two elements are the 2\textsuperscript{nd} and 3\textsuperscript{rd} elements of cv multiplied by 5.
2.1.3 Transporting Vectors

To perform operation with column and row vectors of similar length, it is first needed to transpose the vectors in order to make them either all row vectors or column ones. The apostrophe symbol “ ’ ” is used to convert a column vector to a row one and vise versa. Remember that the length of the vectors must match; otherwise, Matlab will return an error. Experiment with the following examples to learn the use of transpose command.

**Matlab's command:**

```
>> z1 = [2 -1 3], z2 = z1', z3 = z2''
```

**Matlab's response:**

```
z1 =
   2   -1    3
z2 =
   2
   -1
   3
z3 =
   2   -1    3
```

**Comments:**
z1 is a three element row vector. z2 is the transpose of z1. z3 is similar with z1 since after transposing z2 three times is similar as transposing once the z2.

**Matlab's command:**

```
>> z1'+z2, z1+z2
```

**Matlab's response:**

```
ans =
   4
  -2
   6
```

```??? Error using ==> +
Matrix dimensions must agree.
```

**Comments:**
z1’+z2 operation involves the addition of two column vectors, z1+z2 is an illegal operation since a row vector cannot be added with a column vector.

Note that that Matlab can store empty vectors. This is done as follows:
Matlab’s command:
>> Z=[], Z=1:3, Z=[], length(Z)

Matlab’s response:
Z =
[]
Z =
1     2     3
Z =
[]
ans =
0

Comments:
Definition of an empty vector.

Empty vector operation is usually used in cases where the user wants to turn a non-empty vector to an empty one (to reset all the elements of a vector).

2.1.4 Vector Manipulations Related to Products, Division, and Powers
Matlab can perform, scalar products (or inner products) and dot products, as well as dot division and power operations. The only restriction is that the length of the vectors must be the same. The priorities concerning vector manipulations is the same as in the case that we use Matlab as a calculator (power operations first followed by “*” and “/” followed by “+” and “-“)

2.1.4.1 Scalar Product
The scalar product or otherwise termed as inner product, concerns the multiplication of two equal length vectors. The symbol “*” is used to carry out this operation. Given a row vector \( \mathbf{w} \) and a column vector \( \mathbf{u} \) of length \( t \):

\[
\mathbf{\tilde{w}} = [w_1, w_2, ..., w_t], \quad \mathbf{\tilde{u}} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_t \end{bmatrix}
\]
The inner product is defined as the linear combination:

\[ \tilde{w} \tilde{u} = \sum_{i=1}^{t} w_i u_i \]

The following depicts some examples with inner products.

**Matlab’s command:**
>> w=[1 0 2 -1]; u=[2; 4; -2; 0.5];
>> prod1=w*u, prod2=(2+w)*(u/2), prod3=w*w’, prod4=w*u*w*u

**Matlab’s response:**
prod1 =
   -2.5
prod2 =
     3.25
prod3 =
     6
prod4 =
    6.25

**Comments:**
Producing the inner product of various operations. For example prod1 is computed as: prod1=(1*2)+(0*4)+(2*(-2))+((-1)*0.5)=-2.5.

2.1.4.2 Dot Product
The dot product involves the multiplication of two similar vectors (to be either column or row vectors with same length) element-by-element. With the dot product, a new vector is created. The dot product between the row vectors \( \tilde{w} \) and \( \tilde{u}^\top \) (where the superscript \( ^\top \) represents the transpose symbol) is another row vector with the following form:

\[ \tilde{w} \tilde{u}^\top = [w_1 u_1, w_2 u_2, ..., w_2 u_2] \]
The dot product in Matlab is performed via the “.*” symbol. By the use of dot product we can get the inner product. This is explained in the following examples:

**Matlab's command:**
>> dprod1=w.*u', dprod2=u'.*u'.* w

**Matlab's response:**
dprod1 =  
2     0   -4    -0.5

dprod2 =  
4     0    8    -0.25

**Comments:**
Producing the dot product of various operations. For example dprod1 is computed as: dprod1=[1*2, 0*4, 2*(-2), (-1)*0.5].

**Matlab's command:**
>> inner1=sum(w.*u'), inner2=sum((2+w).*u'/2))

**Matlab's response:**
inner1 =  
-2.5

inner2 =  
3.25

**Comments:**
Producing the summation of dot product of various operations. When the sum function is used with a dot product, the result is the inner product of that operation.

In vector manipulations, dot product is performed after dot power (see the following section).

### 2.1.4.3 Dot Division and Power

The dot division does not exist mathematically, but for programming purposes, Matlab includes such operation. The dot division can be used to divide a vector with another vector of similar characteristics and also to divide a vector with a scalar. Through the following examples the use of dot division is outlined. Pay attention on the cases where a dot division produces pathological results.
**Matlab's command:**

```
>> d_div1=-3:1.5:3,  d_div2=1:5
>> d_div2./d_div2, d_div1./d_div2, d_div2./5
```

**Matlab's response:**

```
d_div1 =  
-3.0000  -1.5000       0  1.5000  3.0000

d_div2 =  
1     2     3     4     5

ans =  
1     1     1     1     1

ans =  
-3.0000  -0.7500       0  0.3750  0.6000

ans =  
0.2000  0.4000  0.6000  0.8000  1.0000
```

**Comments:**

Examples of dot division.

---

**Matlab's command:**

```
>>d_div2./d_div1
>> d_div2(3)=0; d_div1./d_div2
>> 1/d_div1
```

**Matlab's response:**

```
Warning: Divide by zero.
(Type “warning off MATLAB:divideByZero” to suppress this warning.)

ans =  
-0.3333  -1.3333       Inf  2.6667  1.6667

Warning: Divide by zero.
(Type “warning off MATLAB:divideByZero” to suppress this warning.)

ans =  
-3.0000  -0.7500       NaN  0.3750  0.6000

??? Error using ==> /
Matrix dimensions must agree.
```

**Comments:**

Examples of pathological results of dot division. In the first case, Matlab returns the result after displaying a warning concerning the division of a number with zero that leads to an infinity quantity. Observe that Matlab returns an Inf for such operations. Likewise, in the second case, the division of 0/0 is not defined.
and a Not-a-Number, NaN, is returned in the vector. In the third case, the inverse of each vector element is required, but Matlab returns an error since for such operation the "./" and not "/" should have been used instead.

**Example:** Examine the following limit:

\[ \lim_{{x \to \infty}} \frac{e^x - e^{-x}}{e^x + e^{-x}} \]

**Matlab’s command:**

```
>> x=[0.5 1 5 25 50 100 300];
>> (exp(x)-exp(-x))./(exp(x)+exp(-x))
```

**Matlab’s response:**

```
ans =
 0.4621   0.7616    0.9999    1.0000    1.0000    1.0000    1.0000
```

**Comments:**

Apparently, the limit converges to unity.

The dot power of vectors works in a similar way as with other dot products. Usually there is the need to square (or to rise to some other power) the elements of a vector. This can be done with the dot power operation as explained in the following example set.

**Matlab’s command:**

```
>> d_power=sqrt(0:log(3):6)
>> d_power.^2, d_power.^4, d_power.* d_power, ans.*ans
```

**Matlab’s response:**

```
d_power =
 0   1.0481   1.4823   1.8154   2.0963   2.3437
ans =
 0   1.0986   2.1972   3.2958   4.3944   5.4931
ans =
 0   1.2069   4.8278  10.8625  19.3112  30.1737
ans =
 0   1.0986   2.1972   3.2958   4.3944   5.4931
ans =
 0   1.2069   4.8278  10.8625  19.3112  30.1737
```
Comments:
Examples of dot power. The second or the forth power of a certain vector can be defined either with the dot power or the dot product operation.

Before leaving this section, the reader should note that dot power of vectors is performed before any other operation, followed by the dot product and the dot division. Of course, if a complex vector command involves manipulations into brackets, these are executed before any other action.

2.1.4.3 Some Useful Vector Functions
Table 2 lists some useful function that can be used with row and column vectors. This list is by no way exhaustive.

<table>
<thead>
<tr>
<th>Basic Information</th>
<th>Elementary Matrices and Arrays</th>
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</thead>
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<td>Display array</td>
</tr>
<tr>
<td>display</td>
<td>Display array</td>
</tr>
<tr>
<td>isempty</td>
<td>True for empty vector</td>
</tr>
<tr>
<td>isequal</td>
<td>True if arrays are identical</td>
</tr>
<tr>
<td>length</td>
<td>Length of vector</td>
</tr>
<tr>
<td>linspace</td>
<td>Generate linearly spaced vectors</td>
</tr>
<tr>
<td>logspace</td>
<td>Generate logarithmically spaced vectors</td>
</tr>
<tr>
<td>ones</td>
<td>Create array of all ones</td>
</tr>
<tr>
<td>rand</td>
<td>Uniformly distributed random numbers and arrays</td>
</tr>
<tr>
<td>zeros</td>
<td>Create array of all zeros</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<tbody>
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<td>Cumulative product</td>
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<tr>
<td>cumsum</td>
<td>Cumulative sum</td>
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<tr>
<td>end</td>
<td>Last index</td>
</tr>
<tr>
<td>find</td>
<td>Find indices of nonzero elements</td>
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<tr>
<td>max</td>
<td>Maximum elements of array</td>
</tr>
<tr>
<td>mean</td>
<td>Average of array</td>
</tr>
<tr>
<td>median</td>
<td>Median of array</td>
</tr>
<tr>
<td>min</td>
<td>Minimum elements of array</td>
</tr>
<tr>
<td>sort</td>
<td>Sort elements in ascending order</td>
</tr>
<tr>
<td>sum</td>
<td>Sum of array elements</td>
</tr>
</tbody>
</table>

Table 2: Various functions that can be used with vectors.

Use the help facilities to see how these functions can be used. The following illustrate the use of some of these functions.

Matlab’s command:
>> clear; clc;
>> x=[]; isempty(x)
>> x=linspace(1, 10, 6), x=logspace(0.1, 0.5, 6); x(7)=0.1; x
>> sort(x), cumsum(x), [Max, l]=max(x), x(end)
Matlab's response:

ans =
1

x =
1.0000 2.8000 4.6000 6.4000 8.2000 10.0000

x =
1.2589 1.5136 1.8197 2.1878 2.6303 3.1623 0.1000

ans =
0.1000 1.2589 1.5136 1.8197 2.1878 2.6303 3.1623

ans =

Max =
3.1623

I =
6

ans =
0.1000

Comments:
Examples with vector functions. The first command instructs for the creation of an empty vector. The isempty function returns 1 for the true statement of an empty vector. Afterwards, linspace is used to generate a row vector of 6 linearly equally spaced points between 1 and 10 whereas logspace is used to generate a row vector of 6 logarithmically equally spaced points between decades 10^0.1 and 10^0.5. "x(7)=0.1" instructs for the creation of a seventh element for the x vector. "sort(x)" sorts the elements of x in ascending order, "cumsum(x)" is a vector containing the cumulative sum of the elements of x. "[Max, I]=max(x)" returns in Max the maximum value of the vector x and in I the index of the maximum element. The last command, "x(end)" returns the last element of the x vector.

2.2 Two Dimensional Arrays (Matrices)
A \( m \times n \) matrix is a rectangular array that has \( m \) rows and \( n \) columns. For example, a 2-by-4 matrix can be the following one:

\[
A_{2\times4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}
\]
It is obvious that the above matrix can be decomposed to either 2 row vectors of $1 \times 4$ dimensions or to 4 column vectors of $2 \times 1$. So, row and column vectors are special cases of a two dimensional array (matrix).

If we let, $a_{11} = 1$, $a_{21} = -2$, $a_{12} = -2$, $a_{22} = 2$, $a_{13} = 5$, $a_{23} = 4$, $a_{12} = -3$ and $a_{24} = 1$, then the “A” matrix is reformed to:

$$A_{2 \times 4} = \begin{bmatrix} 1 & -2 & 5 & -3 \\ -2 & 2 & 4 & 1 \end{bmatrix}$$

The subscripts in each element of the matrix denote the element’s position. For instance, the position of $a_{23}$ is 2$^\text{nd}$ row, 3$^\text{rd}$ column. Matlab stores matrices using this rational. Having in mind that a two dimensional array is a composition of row vectors, we can use spaces (or commas) to define its row vectors and semicolons to separate them. The above matrix can be easily created as follows:

**Matlab's command:**

```matlab
>> A=[ 1 -2 5 -3; -2 2 4 1]
```

**Matlab's response:**

```
A =
   1   2   5   3
  -2   2   4   1
```

**Comments:**

Creation of a 2-by-4 matrix.

To show that row and column vectors are special cases or a two dimensional array (matrix) elaborate on the following examples:

**Matlab's command:**

```matlab
>> B=linspace(1, 4, 5); C=1:5; D=[-4:-1, NaN, Inf 1];
>> A1=[B; C], A2=[A1; A1], A3=[B -9 -5;D]
```

**Matlab's response:**

```matlab
A1 =
   1.0000   1.7500   2.5000   3.2500   4.0000
   1.0000   2.0000   3.0000   4.0000   5.0000
```
A2 =
1.0000 1.7500 2.5000 3.2500 4.0000
1.0000 2.0000 3.0000 4.0000 5.0000
1.0000 1.7500 2.5000 3.2500 4.0000
1.0000 2.0000 3.0000 4.0000 5.0000

A3 =
1.0000 1.7500 2.5000 3.2500 4.0000 -9.0000 -5.0000
-4.0000 -3.0000 -2.0000 -1.0000 NaN Inf 1.0000

Comments:
Various manipulations with matrices.

The examples above, illustrate the case where a larger matrix can be created from smaller ones. Also, pay attention that Matlab returns an error if the user tries to combine row or column vectors with different lengths to create a two dimensional array.

Usually, it is imperative need to store the size of a matrix in some variables. This can be done via a build in function named as size:

Matlab's command:
>> [m1 n1]=size(A1); [m2 n2]=size(A2); [m3 n3]=size(A3);
>> M_N=[m1 n1; m2 n2; m3 n3]

Matlab's response:
M_N =
2  5
4  5
2  7

Comments:
Using the size function to find the size of various matrices. Each row of the M_N matrix contains the size of matrices A1, A2 and A3 respectively.

2.2.1 Transpose of a Matrix
Recall matrix “A” that was defined earlier. The transpose of “A”, symbolized in linear algebra as “A^T” is the following:

\[ A_{2 \times 4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}, \quad A_{4 \times 2}^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{13} & a_{23} \\ a_{14} & a_{24} \end{bmatrix} \]
It is the same idea with the transpose of a row vector to a column one and vise versa. The transpose of an \(m \times n\) matrix is an \(n \times m\) one that is found if the rows of the original matrix become columns in the transposed one. Recall that the transpose in Matlab is performed with “’”. See the following examples to digest this issue.

\begin{itemize}
  \item \textit{Matlab’s command:}
  \[
  >> A4=[1 2 3; 4 5 6], size(A4), A4', size(A4'), A5=[A4' A4']
  \]
  \textit{Matlab’s response:}
  \[
  A4 = \\
  \begin{bmatrix}
  1 & 2 & 3 \\
  4 & 5 & 6
  \end{bmatrix}
  \]
  \[
  \text{ans} = \\
  \begin{bmatrix}
  2 \\
  3
  \end{bmatrix}
  \]
  \[
  \text{ans} = \\
  \begin{bmatrix}
  1 & 4 \\
  2 & 5 \\
  3 & 6
  \end{bmatrix}
  \]
  \[
  \text{ans} = \\
  \begin{bmatrix}
  3 & 2
  \end{bmatrix}
  \]
  \[
  A5 = \\
  \begin{bmatrix}
  1 & 4 & 1 & 4 \\
  2 & 5 & 2 & 5 \\
  3 & 6 & 3 & 6
  \end{bmatrix}
  \]
  \item \textit{Comments:}
  Exploiting the transpose of a matrix.
\end{itemize}

\subsection*{2.2.2 Elaborating on Parts of Matrices – The Colon Notation}
As noted before, in Matlab, a matrix element position is indexed according to the row and column number. Recall the aforementioned “A” matrix:

\[ A_{2 \times 4} = \begin{bmatrix} 1 & -2 & 5 & -3 \\ -2 & 2 & 4 & 1 \end{bmatrix} \]

If someone wants to extract the elements: \(a_{12}, a_{23}, \) and \(a_{24},\) this can be done in the following way (A has already been created and should be stored
in the workspace; use the `who` function to view the variables that you have saved in the workspace or view them from the `workspace browser`:

**Matlab’s command:**

```
>> A(1,2), A(2,3), A(2,4)
```

**Matlab’s response:**

```
ans =  
-2
ans =  
4
ans =  
1
```

**Comments:**

Extracting specific elements from a matrix.

So, in Matlab, to refer to a certain matrix element, after the matrix name the element’s index/position is enclosed in parenthesis. This is similar with the indexing of vectors elements with the exception that in the case of matrices, two indexes should be used (the first one indicates the row number whilst the second the column number). In general, the syntax for manipulating a matrix element in Matlab is:

```
“K(row_index, column_index)”
```

where $K$ is an $m \times n$ matrix, $row\_index$ indicates the row in which an element lays and $column\_index$ the according column. The upper left position in a matrix has (1,1) coordinates. $row\_index$ can take all integer values smaller than $m$, inclusive, while $column\_index$ can also take all integer values smaller than $n$, inclusive (note that Matlab can handle multidimensional arrays just by putting additional indexes to control the higher dimensions).

Moreover, Matlab has an additional way to refer to matrix elements just by using a single index number in the brackets after the declaration of the matrix name. Starting from the upper left element that represents the first element (1) of the matrix, further reference to additional elements is done incrementally one by one and column-wise. That is, the element with
position (2,1) is taken to be the second (2), the (3,1) the third and finally the (m, n) element is the last one (n*m). View the following example to understand this kind of matrix indexing.

Matlab's command:
>> H=magic(4), H(1), H(2), H(9), H(16)

Matlab's response:
H =
16  2  3  13
 5 11 10  8
 9  7  6 12
 4 14 15  1
ans =
16
ans =
5
ans =
3
ans =
1
Comments:
Extracting specific elements from a matrix using single indexing.

For matrices, the colon notation “:” plays an important role. Colon can be used to extract either a specific part of a matrix, or to view a whole row, a whole column or a sub-matrix extracted from the original one. Earlier, the colon was used to define vectors and it has been explained that it creates elements from a starting_value through a finishing_value equally spaced according to a defined step (if the step is not defined, then the step is taken to be 1). In Matlab, a certain row of a matrix can be extracted by placing the colon in the row_index, “K(:,column_index)”, and a certain column by placing the colon in the column_index, “K(row_index,:)”. A sub-matrix that is composed by the r1 to r2 rows (r1=r2) and c1 to c2 columns (c1=c2) of the original matrix can be retrieved according to the following syntax:

“K(r1:r2, c1:c2)”
In addition, there are special cases where the step is defined to be a different integer number other than 1. Some paradigms follow:

**Matlab's command:**

```matlab
>> A(:,1), A(1,:), A(:,1:3), A=[A', A'.*2], A(2:3,3:4)
>> A(1:3,1:2)=A(2:end,3:end)
```

**Matlab's response:**

```matlab
ans =
    1
   -2
ans =
   1   -2    5   -3
ans =
   1   -2    5
   -2    2    4
A =
   1   -2    2   -4
  -2    2   -4    4
   5    4   10    8
  -3    1   -6    2
ans =
  -4    4
  10    8
ans =
  -2   -4
A =
  -4    4    2   -4
  10    8   -4    4
 -6    2   10    8
 -3    1   -6    2
```

**Comments:**
The first command, instructs for the extraction of the first column whilst the second for the extraction of the first row. The following one instructs for the extraction of the 1st, 2nd and 3rd columns. Afterwards, A is re-built; the following command instructs for the extraction of the sub-matrix that is composed from the elements of A that are included in: 2nd and 3rd row – 3rd and 4th columns. The following command is a special case where the step in the colon expression is different than 1. The last command is a more tricky use of the colon.
2.2.2 Matrix Basic Manipulations
Subtraction and multiplication with a scalar are similar as with the vector case. Matrix dimensions (size) must match when performing manipulations with addition and subtraction. Some examples follow:

Matlab's command:

```matlab
>> J=[-1 2 3 1; 5 2 4 2; 1 1 2 0]; I=[1 4 5 2; 8 7 5 1; -1 1 -1 1];
>> J+2*I, J(:,1)+I(:,2), J(1,:)-1/4*I(2,:)
```

Matlab's response:

```
ans =
   1   10   13    5
  21   16   14    4
   -1    3    0    2
```

```plaintext
ans =
   3   12   2
```

```plaintext
ans =
-3.0000  0.2500  1.7500  0.7500
```

Comments:
Basic matrix addition, subtraction and scalar multiplication.

2.1.4 Matrix Manipulations Related to Products, Division, and Powers

Similar rules as with the case of vectors apply to matrix manipulations related with dot operations and products of matrices.

2.1.4.1 Matrix Dot Product, Division and Powers
It is straightforward to generalize the properties of dot operations from the case of vectors to the case of two-dimensional arrays. The operations are again performed element-by-element. Work on the following examples to learn the use of dot operations with matrices (remember which operations come first).
Matlab's command:
>> clear; U=[-1 2 1; 0 -2 3]; V=[1:3; 5 2 4];

Matlab's response:
ans =
1 4 1
0 4 9

ans =
-1 4 3
0 -4 12

ans =
-1.0000 0.5000 0.1111
0 -0.5000 0.1875

ans =
-1.0000 4.0000 1.0000
1.0000 0.2500 27.0000

Comments:
Dot operations with two-dimensional arrays.

2.1.4.2 Matrix to Vector Product – Matrix to Matrix Product
A matrix can be multiplied with a vector or a matrix as long as the following rule concerning their dimensionality/sizes holds:

\[(m \times n) \times (n \times l)\]

That is, the first’s matrix columns equal the second’s rows. A matrix “A” with dimensions \(m \times n\) can be multiplied with a matrix “B” with dimensions \(m \times l\) resulting to a matrix “C” with dimensions \(m \times l\):

\[C_{m \times l} = A_{m \times n} \times B_{m \times l}\]

The above matrix “A” can be multiplied with a \(l \times m\) row vector “R1” from left to produce a \(l \times n\) row vector “R” while “A” can be multiplied with an \(n \times l\) column vector “C1” to the right to produce a column vector “C” with \(m \times l\) dimensions:

\[R_{l \times n} = R1_{l \times m} \times A_{m \times n} \times C1_{n \times l}\]
\[ C_{mxI} = A_{mxI} \times I_{nxI} \]

Let’s define the “A” and “B” matrices:

\[
A_{2\times4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix}, \quad B_{4\times2} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix}
\]

Like linear algebra, Matlab products with matrices (and/or vectors) are calculated as follows:

\[
C_{2\times2} = A_{2\times4} \times B_{4\times2} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \Rightarrow 
C_{2\times2} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} + a_{14}b_{41} & a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32} + a_{14}b_{42} \\ a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31} + a_{24}b_{41} & a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} + a_{24}b_{42} \end{bmatrix}
\]

That is, the element in the (1,1) position of the new matrix “C” if the inner product of the 1st row of “A” with the 1st column of “B”, the (1,2) element of “C” is the inner product of the 1st row of “A” with the 2nd column of “B” and so on. Work through the following examples to digest this topic.

**Matlab’s command:**

```
>> clear; A=[ 1 -2 5 -3; -2 2 4 1];  B=[2 3;1 0;2 -1;1 4]; A_B=[A;B']
>> R1=[-2 -1 3 2]; C1=[1; -1; 2; 0];
>> C=A*B, Row=R1*A_B, Col= A_B*C1, A*(A_B*A_B)*B
```

**Matlab’s response:**

```
C =
  7  -14
  7    -6

Row =
  12   5  -10   16
```
Col =
13
4
5
1

ans =
39  -310
297   314

**Comments:**
Matrix-vector and matrix-matrix products.

### 2.1.5 Special Cases of Matrices

Matlab has some very useful *build-in* matrices *functions* that ease the manipulation of two dimensional arrays. These *functions* usually take as input (at least) the matrix dimension and return a certain result. Table 3 exhibits these very useful functions accompanied with a short explanation of their use.

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>eye(m,n)</code></td>
<td>In the case where <code>m</code> is equal to <code>n</code> this returns an identity matrix</td>
</tr>
<tr>
<td><code>ones(m,n)</code></td>
<td>Creates an <code>m-by-n</code> matrix of ones</td>
</tr>
<tr>
<td><code>diag(V)</code></td>
<td>When <code>V</code> is a <code>m-by-m</code> matrix returns the elements of the main diagonal.</td>
</tr>
<tr>
<td><code>rand(m,n)</code></td>
<td>Creates an <code>m-by-n</code> matrix with random entries</td>
</tr>
<tr>
<td><code>zeros(m,n)</code></td>
<td>Creates an <code>m-by-n</code> matrix of zeros</td>
</tr>
</tbody>
</table>

**Table 3:** Various functions that can be used with vectors.

Examples with the above matrix-functions follow:

**Matlab's command:**

```matlab
>> eye(3,2), ones(2,3), diag([2 3; 1 -2]), rand(2,2), zeros(1,2)
```

**Matlab's response:**

```matlab
ans =
1 0
0 1
0 0

ans =
1 1 1
1 1 1

ans =
2
-2
```

47
ans =
0.9501  0.6068
0.2311  0.4860

ans =
0  0

Comments:
Examples with matrices build-in functions.

2.1.5 Additional Useful Matrix Functions
Functions that are exhibited in Table 1 can also been applied to a two
dimensional arrays because vectors are special cases of matrices (and vise
versa). Additional build-in functions are illustrated in Table 4.

<table>
<thead>
<tr>
<th>Basic Information</th>
<th>Elementary Matrices and Arrays</th>
</tr>
</thead>
<tbody>
<tr>
<td>ndims</td>
<td>Number of dimensions</td>
</tr>
<tr>
<td>numel</td>
<td>Number of elements</td>
</tr>
<tr>
<td>blkdiag</td>
<td>Block diagonal concatenation</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operations and Manipulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>det</td>
</tr>
<tr>
<td>expm</td>
</tr>
<tr>
<td>flip</td>
</tr>
<tr>
<td>flipud</td>
</tr>
<tr>
<td>inv</td>
</tr>
<tr>
<td>logm</td>
</tr>
<tr>
<td>norm</td>
</tr>
<tr>
<td>rank</td>
</tr>
<tr>
<td>sortrows</td>
</tr>
<tr>
<td>sqrtm</td>
</tr>
<tr>
<td>trace</td>
</tr>
<tr>
<td>triu</td>
</tr>
<tr>
<td>triu</td>
</tr>
</tbody>
</table>

Table 4: Various functions that can be used with vectors.

Moreover, type: “help matfun” to view some very useful matrix functions
related with numerical linear algebra.

2.1.6 Example: System of Linear Equations
Many times, a researcher has to solve a system of linear equation
simultaneously. In matrix notation, such problem is formulated as:

$$Ax = b$$

where “A” is a coefficient matrix, “x” is the set (column) of unknowns and “b”
is a column vector. In an equation-wise format, the above is equal to:
The solution of the above system can be expressed as:

\[ x = A^{-1}b \]

given that \( A^{-1} \) (that represents the inverse matrix of \( A \)) exists and the number of equation, \( n \), is equal or greater than the number of unknowns, \( m \).

A unique solution to the above equation system exists when: \( n=m \).

Matlab has standard and efficient specialized routines to solve such systems like the mldivide function (called alternatively with \( \backslash \)). Otherwise, a quick way (but some times non practical and inaccurate if the number of equations is extremely large) to solve the above system is via the inverse function, inv (note that the inv can be used only with square matrices). View the following example to understand how you can solve a small system of linear equations:

**Matlab’s command:**

```
>> clear; A=[2 4 3; -2 -4 2; 6 4 2]; b=[2; 4; 1];
>> x1=inv(A)*b, x2=A\b
```

**Matlab’s response:**

```
x1 =
0.0500
-0.4250
1.2000

x2 =
0.0500
-0.4250
1.2000
```

**Comments:**

Two ways for solving a system of linear equations.
To see that these two procedures lead to different results, elaborate on the following example (search online help to see the use of the command: pascal):

**Matlab's command:**

```matlab
>> clear; A=pascal(10); b=[5; -2; 4; 8; 1; 2; 0; -6; 2; 3];
>> x1=inv(A)*b;, x2=A\b;
>>diff=x1-x2
```

**Matlab's response:**

```
diff =
   -1.0328 e-006
   9.0204 e-006
  -3.4539 e-005
   7.6293 e-005
  -0.00010781
   0.0001013
 -6.3407 e-005
  -2.5521 e-005
 -5.9979 e-006
   6.2738 e-007
```

**Comments:**

For even moderate linear systems, the two methods (inv and “\”) lead to different solutions. If high precision is required, then the researcher should prefer the “\”.

The most proper way for large systems is the use of “\”. See the help facilities for details on inv and on “\”.
3. Plots and Graphs (2D and 3D)

It is quite easy to create a plot or a graph by using the variables or parameters that have already been stored in the Matlab's \textit{workspace}. Matlab offers a variety of \textit{build-in functions} for creating simple two dimensional plots, 3 dimensional surface plots, to combine plots to a larger one and so on. The following subsections are a very brief and an elementary reference to the visualization capabilities of Matlab.

3.1 Creating 2D Line Plots

The basic function for the creation of a simple 2D line plot is called: \texttt{plot}. Let be a row or column vector with real data named \texttt{Y}, with elements "$y_1, y_2, ..., y_n"$. If \texttt{plot} is called as: "\texttt{plot(Y)}", then a linear plot of the elements of \texttt{Y} versus its index will appear (the \texttt{plot} function creates/plots the pairs: $(1, y_1), (2, y_2), ..., (n, y_n)$ and connects them with a line). If for each "$y_i$" we have an accompanied "$x_j$" coordinate, then the function "\texttt{plot(X,Y)}" plots all pairs: $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$ and connects them with a line. Note that vectors \texttt{Y} and \texttt{X} must share the same length. Additionally, if the data to be plotted are stored in a two dimensional array, then the arguments for the \texttt{plot} \textit{function} can be done via the use of colon notation ":". It is very easy to plot the following function in the range $[0,5]$:

$$y = \frac{2x^2 + 5x - 1}{x^3 - 5}$$

\textit{Matlab's command:}

\begin{verbatim}
>> clear; x=0:0.5:5; y=(2*x.^2+5*x-1)./(x.^3-5); plot(x,y);
>> x=0:0.2:5; ; y=(2*x.^2+5*x-1)./(x.^3-5); plot(x,y);
\end{verbatim}
**Matlab's response:**

![Graph](image)

**Comments:**
Plot of a function. The use of dot operations is prominent for this task. Note that the plot's smoothness is controlled by the density of the x vector. In the second plot where the x is denser, the function plot is smoother.

### 3.1.1 Plot Edit Mode

If you have already type the aforementioned commands on the Matlab command window, you will have noticed that the plots that you see, appear in a figure window (i.e Figure 10) that offers a variety of tools that allow you to edit and customize the plot under consideration by using a point and click style editing mode.
This figure window has tools that look like a photo editor (e.g. Microsoft® Photo Editor). You can select objects in a graph, cut, copy, paste, move and resize objects, zoom in and zoom out at certain areas of the plot, change the settings of the axes, print or save the figure for a later use or even to export the plot in a certain format (e.g. *.emf, *.bmp, *.jpg, *.tif, etc). Create a figure and play around with the various tools to learn their use.

The plots that are included in this handout (except the ones that were copied from [2]) have been exported from the figure window in an enhanced metafile format (*.emf). To export a plot, from the figure window go: File>Export, and from the window that appears choose the exporting format, give a name to the file and save it in a folder of your choice. By doing this, you can easily insert plots from Matlab to a text editor like Microsoft® Word.

3.1.2 Functions and Style Facilities Related to Plots
The blue solid line is the default style for the plots. This of course can change. Matlab gives the user the option to define a different line color and
to mark each plotted point with a symbol. To do this, the user should call
the plot function as:

```
plot(X,Y, '???')
```

where “?” represents the color, “?” the mark style and “?” the line style (if
not given, then Matlab uses a default style). Table 5 tabulates the plot
facilities (see also the online help).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Color (?)</th>
<th>Symbol</th>
<th>Line Style (?)</th>
</tr>
</thead>
<tbody>
<tr>
<td>g</td>
<td>Green</td>
<td>, ; , . , --</td>
<td>Solid, dotted, dash dot and dashed</td>
</tr>
<tr>
<td>r</td>
<td>Red</td>
<td>-. ;, ., -</td>
<td>Solid, dotted, dash dot and dashed</td>
</tr>
<tr>
<td>c</td>
<td>Cyan</td>
<td>- ; , ., -</td>
<td>Solid, dotted, dash dot and dashed</td>
</tr>
<tr>
<td>m</td>
<td>Magenta</td>
<td>-. , , x</td>
<td>Point, circle and x-mark</td>
</tr>
<tr>
<td>y</td>
<td>Yellow</td>
<td>+ ; , , s</td>
<td>Plus, star and square</td>
</tr>
<tr>
<td>k</td>
<td>Black</td>
<td>d ; , v</td>
<td>Diamond and triangle down</td>
</tr>
<tr>
<td></td>
<td></td>
<td>^ ; , &lt;, &gt;</td>
<td>Triangles: up, left and right</td>
</tr>
<tr>
<td></td>
<td></td>
<td>p ; , h</td>
<td>Pentagram, hexagram and solid</td>
</tr>
</tbody>
</table>

Table 5: Plot colour and line styles.

Moreover, Table 6 lists additional commands that are useful when using the
plot function.

<table>
<thead>
<tr>
<th>Function</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>axis</td>
<td>Sets the minimum and maximum values of axes.</td>
</tr>
<tr>
<td>clf</td>
<td>Clear the current figure.</td>
</tr>
<tr>
<td>close (all)</td>
<td>Closes the current (all) figure(s).</td>
</tr>
<tr>
<td>figure</td>
<td>Creates a new figure window.</td>
</tr>
<tr>
<td>grid on/off</td>
<td>Adds/removed major grid lines to the current axes.</td>
</tr>
<tr>
<td>gtext('text')</td>
<td>Adds text to a figure manually, by the use of mouse.</td>
</tr>
<tr>
<td>hold on/off</td>
<td>hold on/off holds the current plot and all axis properties so that</td>
</tr>
<tr>
<td></td>
<td>subsequent graphing commands add to the existing graph.</td>
</tr>
<tr>
<td>legend('text')</td>
<td>Adds (removes) a legend to the graph.</td>
</tr>
<tr>
<td>legend off</td>
<td>The same as with plot but logarithmic scale axes are used.</td>
</tr>
<tr>
<td>loglog</td>
<td>The same as with plot with the exception that logarithmic scale is used for X-axis.</td>
</tr>
<tr>
<td></td>
<td>The same as with plot with the exception that logarithmic scale is used for Y-axis.</td>
</tr>
<tr>
<td>subplot</td>
<td>Breaks the current figure to subplots.</td>
</tr>
<tr>
<td>title('text')</td>
<td>Adds a title above the figure.</td>
</tr>
<tr>
<td>xlabel('text')</td>
<td>X-axis label</td>
</tr>
<tr>
<td>ylabel('text')</td>
<td>Y-axis label</td>
</tr>
</tbody>
</table>

Table 6: Various functions that help in the creation of 2D-plots.
View the following examples to digest the plot facilities.

**Matlab's command:**

```matlab
>> plot(x, y, 'md: '); figure; plot(x, y, 'r*--'); hold on;
>> y1=rand(1,length(y)); plot(x,y1, 'bo'); plot(x,y1, 'r+');
>> xlabel('X-values'); ylabel('Y-values'); title('This is a plot');
>> close all; plot(x,y, 'bp--', x, log(y1.^2), 'rh', x, log(y1.^2),'bs');
```

**Matlab's response:**

![Plot 1](image1.png)

![Plot 2](image2.png)
**Comments:**
Experimenting with various functions and facilities related with the plots. The last *plot* command shows how few plots can be depicted in one figure without using the hold on command.

The following examples, concern the creation of some subplots in one *figure window*. If you view the online help for the *subplot* command, you will see that it takes three input arguments. That is, it is called as:

```
"subplot(m,n,p)"
```

It breaks the *figure window* into an *m-by-n* matrix of small axes, selects the *p*-th axes for the current plot, and returns the axis handle. The axes are counted along the top row of the *figure window*, then the second row, etc. The *subplot* function instructs which subplot is handled at the current time. After the *subplot* command, a *plot* command should follow. See the following examples:

**Matlab’s command:**
```
>> close all; subplot(2,3,1); plot(x,y, 'r--'); title('Subplot 1');
>> subplot(2,3,2); plot(x,y1, 'y*'); title('Subplot 2');
>> subplot(2,3,3);
>> subplot(2,3,5); plot(randn(1,30), 'bo:'); title('Subplot 5');
>> subplot(2,3,6); plot(x,y, 'bp--', x, (y1.^2), 'rh', x, log(y1.^2),'bs');
>> title('Subplot 6');
```
**Matlab's response:**

![Subplot images](image-url)

**Comments:**
Experimenting with the `subplot` command. The first two input arguments to the `subplot` function define the dimensionality of the *figure window*. If `m` is the first input arguments and `n` the second, then the *figure window* breaks to an *m-by-n* plot matrix. The third input to the `subplot` function designates the current subplot to be handled. Starting from the upper left corner and moving row-wise to the lower right corner, 1 refers to the first subplot, 2 to the second and *mn* to the last one. After each `subplot` command, the `plot` command should follow. In the current examples, the commands instruct for the creation of a 2-by-3 subplot matrix. The 1st, 2nd, 5th and 6th subplots are used whilst the 3rd and 4th are currently unused. Note that if the `subplot` command is avoided for any subplot, then its space on the figure window stays empty. If only the `subplot` is called as with the case of the third subfigure, then a blank figure appears in the *figure window*.

Moreover, Matlab offers a variety of options to customize the graph. For example, a more general calling syntax for the `plot` is:

```
plot(X, Y, '??', 'PropertyName', PropertyValue, ...)
```

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where “PropertyName” relates with an element of the plot and “PropertyValue” is defined by the user. Some useful properties concern the following:

- “LineWidth” - specifies the width (in points) of the line.
- “MarkerEdgeColor” - specifies the color of the marker or the edge color for filled markers (circle, square, diamond, pentagram, hexagram, and the four triangles).
- “MarkerFaceColor” - specifies the color of the face of filled markers.
- “MarkerSize” - specifies the size of the marker in units of points.

View the following examples to conceptualize this issue:

**Matlab’s command:**

```matlab
>> plot(x,y,'bd-.', 'LineWidth', 2, 'MarkerSize', 11, ... 
    'MarkerFaceColor', 'r', 'MarkerEdgeColor', 'g')
```

**Matlab’s response:**

![Plot Example](image_url)

**Comments:**
An example of a plot that uses some of the plot properties.

Besides these 2D visualization facilities, Matlab can be used to create bar charts, pie charts, histograms, stems plot, stairstep plots, errorbars etc. Use the online help and the *help browser* to see how you can use these visualizations facilities and learn more about the plot properties.
3.2 Creating 3D Graphs
Matlab offers fancy 3-dimensional line plots and surface graphs.

3.2.1 Creation of 3D Line Plots
There is the function `plot3` which its use is similar as with the function `plot`, with the difference that an additional input argument corresponding to the additional dimension is required (the additional dimension is symbolized as “z”). `plot3` is used to plot lines or point coordinates in the 3D space. Let use the `plot3` to create a 3D helix:

Matlab's command:
```matlab
>> clear; g=linspace(-5*pi,5*pi,200); plot3(sin(g),cos(g),g,'r*-');
>> xlabel('x'); ylabel('y'); zlabel('z'); title('A helix');
```
Matlab's response:
![3D Helix Graph]

Comments:
Creating a helix with the `plot3` function.

3.2.2 Creation of 3D Mesh and Surface Graphs
There are also many commands that lead to the creation of 3D surfaces. Recall that to create a surface of two variables, at each pair of \((x,y)\) a
The functional expression, $z=f(x,y)$, is evaluated. Usually, we evaluate the $z$ behavior in a certain area of the $(x,y)$-plane in which $z$ has some interesting properties (e.g. it presents a minimum, a maximum or a saddle point). Suppose that a vector that defines the $x$-axis (abscissa) and a vector that defines the $y$-axis (ordinate) exist and define the $(x,y)$-plane on which we would like to create the 3D surface of $z=f(x,y)$. To plot the surface, it is needed to create a grid of sample points (most preferable with high density and this is defined by the difference between the elements of $x$ and $y$) that covers the rectangular domain of the $(x,y)$ plane in order to generate $X$ and $Y$ matrices consisting of repeated rows and columns of $x$ and $y$, respectively, over the domain of the function. Then these matrices will be used to evaluate and graph the function.

We will see how we can plot a 3D surface via an example. Let say that we need a graph of the well know peaks function that has the following functional form:

$$Z = f(x,y) = 3(1-x)^2 e^{-x^2-(y+1)^2} - 10\left(\frac{x}{5} - x^3 - y^5\right) e^{-x^2-y^2} - \frac{1}{3} e^{-(x+1)^2+y^2}$$

We want to examine this function in the rectangular space of $(x,y)$-plane defined as:

$$-2 \leq x \leq 2$$
$$-4 \leq y \leq 4$$

Since Matlab understands combinations of $(x,y,z)$ pairs only, it is needed to evaluate $Z$ at various points (the number of these points defines the surface smoothness) that lay in the above rectangular space. meshgrid can be used to create the desired grid of points. The following code lead to the generation of the grid:

```matlab
Z = f(x,y) = 3(1-x)^2 e^{-x^2-(y+1)^2} - 10(\frac{x}{5} - x^3 - y^5) e^{-x^2-y^2} - \frac{1}{3} e^{-(x+1)^2+y^2}
```
Matlab's command:

```matlab
>> clear; x=-2:1:2; y=-4:1:4; [X,Y]=meshgrid(x,y)
>> plot(X,Y, 'rh'); axis([-3 3 -5 5]); xlabel('x-axis'); ylabel('y-axis');
>> title('Meshgrid');
```

Matlab's response:

```
X =
  -2  -1   0   1   2
  -2  -1   0   1   2
  -2  -1   0   1   2
  -2  -1   0   1   2
  -2  -1   0   1   2
  -2  -1   0   1   2
  -2  -1   0   1   2
  -2  -1   0   1   2

Y =
  -4  -4  -4  -4  -4
  -3  -3  -3  -3  -3
  -2  -2  -2  -2  -2
  -1  -1  -1  -1  -1
   0   0   0   0   0
   1   1   1   1   1
   2   2   2   2   2
   3   3   3   3   3
   4   4   4   4   4
```

Comments:
Creating the grid necessary for plotting a 3D surface. **meshgrid**
takes as inputs the vectors x and y that define the (x,y)-plane that
should be used to evaluate Z and returns two matrices (X and Y) that
if taken together, generate various (x,y) pairs that lay in the
rectangular domain. The subsequent 2D plot of X and Y shows how the grid is sampled.

Now, we should evaluate the Z at each pair of the grid and generate the surface plot. This is done with the following code:

**Matlab's command:**

```
>> Z = 3*(1-X).^2.*exp(-(X.^2) - (Y+1).^2) - ... 
   10*(X/5-X.^3-Y.^5).*exp(-X.^2-Y.^2) - 1/3*exp(-(X+1).^2-Y.^2);
>> figure(1); mesh(X,Y,Z); xlabel('x'); ylabel('y'); zlabel('Z');
>> figure(2); surf(X,Y,Z); xlabel('x'); ylabel('y'); zlabel('Z');
```

**Matlab's response:**
Comments:
Creating the 3D mesh and a 3D surface. The function is not smooth because the dense of the x and y was too low.

With the above code, a 3D mesh and a 3D surface is created. It is apparent that the surfaces are not smooth because the original density of x and y vectors was too low. In the following, we re-create the mesh and surface with higher densities.

Matlab’s command:
```matlab
>> x=-2:0.25:2; y=-4:0.5:4; [X,Y]=meshgrid(x,y);
>> Z =3*(1-X).^2.*exp(-(X.^2) - (Y+1).^2)- 10*(X/5-X.^3-Y.^5).*exp(-X.^2-Y.^2)-1/3*exp(-(X+1).^2-Y.^2);
>> figure(1); mesh(X,Y,Z); xlabel('x'); ylabel('y'); zlabel('Z'); title('Peaks')
>> figure(2); surf(X,Y,Z); xlabel('x'); ylabel('y'); zlabel('Z'); title('Peaks');
```
Matlab's response:

Comments:
Creating the 3D mesh and a 3D surface with a higher density meshgrid.
To find what other useful commands can be accompanied the `mesh` and `surf` commands, use the online help.

### 3.2.2.1 Examination of a Function’s Critical Points

Many times, a researcher is faced with an unconstrained optimization problem. Than is, it has a two variable function $Z=f(x,y)$ and wants to examine this function in a certain area in order to locate any minimum, maximum or saddle point(s). From the previous section, it is apparent that in the range $[-10,10]$ the peaks function seems to have various critical points. In optimization theory, a critical point can be located via the use of a contour plot. A contour plot depicts the level curves of $f(x,y)$ for some values $V$. In other words, we project the sections/incision of various heights of the $Z=f(x,y)$ function in the $(x,y)$-plane. Matlab offers such `function` named as: `contour`. Experiment with the following code to see the usefulness of contour plots.

**Matlab’s command:**

```matlab
>> V=-10:1:10; [c,h] = contour(x,y,Z,V); clabel(c,h), colorbar; hold on;
>> xlabel('x'); ylabel('y'); title('Contour plot of the peaks function');
```

**Matlab’s response:**

![Contour plot of the peaks function](image.png)
**Comments:**
Creating the contour plot of the peaks function. Viewing this plot, it is easy to identify the critical points of the function. There is one global maximum, one global minimum, two local maxima and a local minimum. Additionally, from the color-bar that is created with the command: colorbar, and from the values printed next to the contour curves, it is obvious that the peaks functions around its critical points exhibits a flatness (saddle points) (Note: the “*” that indicate a critical point are plotted approximately); the text next to each critical point was placed with the gtext command.

Alternatively, we can approximate the maximum of the peaks function by searching exhaustively the Z matrix (to do so, we should re-define the x and y vectors to have high dense). This is done with the following code:

**Matlab's command:**
>> close all; x=-2:0.05:2; y=-4:0.25:4; [X,Y]=meshgrid(x,y);
>> Z =3*(1-X).^2.*exp(-(X.^2) - (Y+1).^2)-...
10*(X/5-X.^3-Y.^5).*exp(-X.^2-Y.^2)-1/3*exp(-(X+1).^2-Y.^2);
>> V=-10:1:10; [c,h] = contour(x,y,Z,V); clabel(c,h), colorbar; hold on
>> xlabel('x'); ylabel('y'); title('Contour plot of the peaks function');
>> Fmax=max(max(Z)); P=find(Z==Fmax); xy=[X(P) Y(P)];
>> plot(xy(1),xy(2), 'rp', 'MarkerSize',10);
>> text(xy(1)+0.05,xy(2), 'Maximum Point');

**Matlab's response:**

![Contour plot of the peaks function](image-url)
**Comments:**
Locating the maximum of the peak function via an exhaustive search. “Fmax=max(max(Z))” stores in Fmax the maximum value of the Z matrix that is equal to 7.9966. “P=find(Z==Fmax)” returns in P the element index (a single value) where Z is equal to Fmax specifically the Z element with index number 1343 has the value of 7.9966. Finally, in xy we store the x-and-y coordinates of the maximum point. “MarkerSize” in the plot command creates a larger size mark symbol.

For practice try to write your own code to find the global minimum of the peaks function (you can also try to find the local minima and maxima but this is tricky).
4. Control Flow
A collection of commands that include repetition of a conditional code segment is termed as *control flow* structure. In Matlab as in any programming language, *control flow* commands are presented via the execution of a conditional expression (e.g. execute a code segment if a statement is true) that is based on logical and/or relational operators.

4.1. Logical and Relational Operators
To evaluate a conditional code, it is needed to have a Boolean expression that takes two possible values: TRUE or FALSE. Matlab defines a TRUE statement with the value of 1 and a FALSE with the value of 0. In Matlab there are six relational operators: “<” (less than), “<=" (less than or equal to), “>” (greater than), “>=" (greater than or equal to), “==” (equal to), and “~=“ (not equal to). These can be used to compare a variable to a scalar, a vector to a vector or a multidimensional array to another one. Always the comparisons are done element by element and the result is a scalar/vector/matrix of the same size as then ones originally used, with elements set to one where the relation is true and elements set to zero where it is not.

In addition, Matlab has (at least) 3 logical operators: “&” (logical AND), “|” (logical OR), and “~” (logical complement - not). If we have two matrices “A” and “B” that share the same dimensionality then:

- “A” & “B” is a matrix whose elements are 1’s where both “A” and “B” have non-zero elements, and 0’s where either has a zero element. “A” and “B” must have the same dimensions (or one can be a scalar). For example in the case of scalars: 1 & 0, 0 & 1 and 0 & 0 will result to 0 (FALSE) while 1 & 1 is 1 (TRUE).
- “A” | “B” is a matrix whose elements are 1’s where both “A” or “B” has a non-zero element and 0’s where both have zero elements. “A” and “B” must have the same dimensions (or one can be a scalar). For
example and for the case of a scalar: \(0 | 1, 1 | 0, \text{ and } 1 | 1\) is 1 (TRUE) while \(0 | 0\) is 0 (FALSE).

- “\(\sim A\)” is a matrix whose elements are 1’s where “\(A\)” has zero elements and 0’s where “\(A\)” has non-zero elements. For example and in the case of a scalar: \(\sim 1\) is (FALSE) and \(\sim 0\) is 1 (TRUE).

Note that for Matlab, any number (integer or real, positive or negative) different from 0 (zero) is a true statement, that is, it has the same meaning as with 1. Pay also attention that the precedence with relational and logical operators is the following (from highest to lowest):

1. Transpose “\(.'\)”, power “\(.^\)” , complex conjugate transpose “\(.'\)” , matrix power “\(^\)”;  
2. Unary plus “\(+\)”, unary minus “\(-\)”, logical negation “\(\sim\)”;
3. Multiplication “\(.*\)” , right division “\(/\)” , left division that returns the inverse of a division “\(./\)” , matrix multiplication “\(^\)” , matrix right division “\(./\)” , matrix left division “\(\backslash\)”;
4. Addition “\(+\)”, subtraction “\(-\)”;
5. Colon operator “\(:\)”;  
6. Less than “\(<\)” , less than or equal to “\(<=\)” , greater than “\(>\)” , greater than or equal to “\(>=\)” , equal to “\(==\)” , not equal to “\(\sim=\)”;
7. Element-wise logical AND “\&\)”;
8. Element-wise logical OR “\(|\)”.

View the examples that follow to digest the use or logical and relational operators.

**Matlab’s command:**

```matlab
>> clear all; x=2; y=0; z=-2;
>> a=(x>=1), b=(x~1 & y), c=(1 | y & x>0), d=(~y & z | ~x == 5)
```

**Matlab’s response:**

```matlab
a =
   1
b =
   0
c =
   1
```
Logical and relational operations. \( a \) is 1 because the statement evaluated is TRUE since \( x \) is 2 and greater than 1. \( b \) is FALSE because: “\( x\neq 1 \)” returns 1, and “1 \& y” returns 0. \( c \) is TRUE because: “\( x>0 \)” is 1, and “1 | y \& 1” is “1 | 1” which is TRUE. \( d \) is TRUE because: “\( \sim y \)” is 1 and “\( \sim x \)” is 0 so we have “(1 \& z | 0==5)” that is equal to “(1 \& z | 0)” that is equal to: “1 | 0” that is TRUE.

Some examples with matrices and the logical and relational operators follow:

\[
\text{Matlab's command:}\\
\text{\texttt{clear all; x=[5 0 -1; 2 4 8]; y=[2 1 -1; 1 2 4];}}\\
\text{\texttt{a=(x==5), b=(x~=1 \& y), c=(2*y==x | \sim x), d=(x>0 | y==1 \mid x=x)}}\\
\text{Matlab's response:}\\
a = \\
1 0 0 \\
0 0 0 \\
b = \\
1 1 1 \\
1 1 1 \\
c = \\
0 1 0 \\
1 1 1 \\
d = \\
1 1 0 \\
1 1 1
\]

Comments: Logical and relational operations with matrices. In \( a \) we have only the first element to be 1 since it the only one that is equal to 5. We get \( b \) to be a TRUE matrix because “\( x\neq 1 \)” returns a matrix of 1’s since none element is equal to 1. By definition, all elements in \( y \) are taken to be TRUE statements. So TRUE \& TRUE gives \( b \) full of 1’s (Figure out how the \( c \) and \( d \) were evaluated).

4.1.1 The any and all Functions
Matlab has two build-in functions that are work as logical operators. The first one is the any function that is TRUE if any element of a vector is nonzero. For vectors, any returns 1 if any of the elements of the vector are non-zero. Otherwise it returns 0. For matrices, any operates on the columns of the
matrix, returning a row vector of 1’s and 0’s. The second one, all, becomes TRUE if all elements of a vector are nonzero. For vectors, all returns 1 if none of the elements of the vector are zero. Otherwise it returns 0. For matrices, all operates on the columns of the matrix, returning a row vector of 1’s and 0’s. View the following code to understand these two functions.

**Matlab’s command:**

```matlab
>> clear all; z=[-5 2 0]; x=[5 0 -1; 2 4 8]; y=[2 1 -1; 1 2 4];
>> a1=any(z), a2=any(x), a3=any(y), a4=all(z), a5=all(x), a6=all(y)
```

**Matlab’s response:**

```
a1 = 
    1

a2 =
    1    1    1

a3 =
    1    1    1

a4 =
    0

a5 =
    1    0    1

a6 =
    1    1    1
```

**Comments:**

Displaying the use of any and all.

**4.2. Conditional Statements**

The above section has been introduced since it is necessary for the evaluation of conditional statements as well as loop expressions (see the following section). A conditional statement is a segment of programming code that evaluates a statement; if the statement is TRUE then it executes some commands, otherwise if it is FALSE it runs a bulk of different programming code. The two most important conditional statements that Matlab materialize, is the if statement and the switch statement.
4.2.1 The if Statement

The if statement evaluates a logical expression and executes a group of statements when the expression is TRUE. The optional elseif and else keywords can be used for the execution of alternate groups of statements. An end keyword, which matches the if, terminates the last group of statements. The groups of statements are delineated by the four above keywords-no braces or brackets are involved [2]. The three possible syntax versions of this conditional expression are tabulated in Table 7.

<table>
<thead>
<tr>
<th>Syntax</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>if logical expression segment of programming code executed if the logical expression is TRUE end</td>
<td>flag=1; if ~isempty(flag) disp('Hello') end sales=5000; if sales&lt;1000 Profit=sales<em>0.1; else Profit=(sales-1000)<em>0.2+1000</em>0.1; end sales=5000; if (sales&gt;1000 &amp; sales&lt;=2000) disp('Low Sales'); Profit=sale</em>0.1; elseif (sales&gt;2000 &amp; sales&lt;=10000) disp('Medium Sales'); Profit=sales<em>0.15; else disp('Satisfactory Sales'); Profit=sales</em>0.17; end</td>
</tr>
<tr>
<td>elseif logical expression #2 segment of programming code executed if the logical expression #2 is TRUE else segment of programming code executed if the previous logical expressions are FALSE end</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: The if statement syntax and examples.
Although it is possible to run these examples via the *command window*, it is better to use the Matlab *Editor/Debugger* to write a script instead. Real examples with the conditional statements as well as with the loops are postponed until the introduction of scripts in section 5.

### 4.2.2 The **switch** Statement

The **switch** statement executes groups of statements based on the value of a variable or expression. The keywords **case** and **otherwise** delineate the groups. Only the first matching case is executed. There must always be an end to match the **switch**. [2]. The expression following the **case** should be either a scalar or a string. The syntax of this conditional expression is tabulated in Table 8.

<table>
<thead>
<tr>
<th>Syntax</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>switch</strong> expression</td>
<td><code>dice=3;</code></td>
</tr>
<tr>
<td><strong>case</strong> choice #1</td>
<td><code>switch</code> (dice)</td>
</tr>
<tr>
<td><em>segment of executable programming code</em></td>
<td><code>case 1</code> disp('One')</td>
</tr>
<tr>
<td></td>
<td><code>case 2</code> disp('Two')</td>
</tr>
<tr>
<td><strong>case</strong> choice #2</td>
<td><code>case 3</code> disp('Three')</td>
</tr>
<tr>
<td><em>segment of executable programming code</em></td>
<td><code>case 4</code> disp('Five')</td>
</tr>
<tr>
<td><strong>otherwise</strong></td>
<td><code>case 5</code> disp('Five')</td>
</tr>
<tr>
<td><em>segment of executable programming code</em></td>
<td><code>otherwise</code> disp('Six')</td>
</tr>
<tr>
<td><strong>end</strong></td>
<td><code>end</code></td>
</tr>
</tbody>
</table>

**Table 8:** The **switch** statement syntax and example.

### 4.3. Loop Expressions

As mentioned before, loop statements are useful when a segment of executable programming code is needed to be executed either a specific number of times, than or as long as an expression is TRUE. The **for** and **while** loop statements are needed commands when writing efficient programming code.
4.3.1. The for Loop
The for loop repeats a group of statements a fixed, predetermined number of times. A matching end delineates the statements. The syntax for the for loop is exhibited in Table 9.

<table>
<thead>
<tr>
<th>Syntax</th>
<th>Example</th>
</tr>
</thead>
</table>
| *for* index=first_value:step:last_value  
  *segment of executable programming code*  
  *end* | *for* *i*=1:10  
  *disp(i)*  
  *
end*  
  *sumj=0;*  
  *for* *j*=25:-2:-12  
  *sumj=sumj+j;*  
  *
end* |

*Table 9:* The for statement syntax and example.

The colon notation is similar as in the case of the vectors. Actually, *index* in the for syntax is a vector with *n* elements with first element being the *first_value* and last the *last_value*. The difference between the *index* elements is *step*. Afterwards, the for statement executes *n* times the segment of executable programming code moving from the first element of the *index* to the last one element-by-element at each time. If *step* is not displayed, then by default is set to 1.

4.3.2. The while Loop
The while loop repeats a group of statements an indefinite number of times under control of a logical condition. That is, as long as an expression is TRUE, then the segment of executable programming code that is included in the while statement is executed. A matching end delineates the statements [2]. The syntax for the while loop is exhibited in Table 10.

<table>
<thead>
<tr>
<th>Syntax</th>
<th>Example</th>
</tr>
</thead>
</table>
| *while* expression  
  *segment of executable programming code*  
  *end* | *X=3;*  
  *while* *X*<=10  
  *disp(X)*  
  *X=X+1;*  
  *
end* |

*Table 10:* The while statement syntax and example.
4.3.3. Nested Conditional and Loop Expressions

All conditional and loop statements can be combined in any way that is desirable. If for example a loop expression includes an if statement that nests a switch statement and so on, then, in programming terminology, it is termed that we have nested statements. As long as the statements are nested in a logical and a non-redundant manner, the user will get correct answers. View the following example that is exhibited in Table 11 and nests various statements (figure out their use).

An example of syntax of nested conditional and loop statements

```matlab
clear; flag=1; x=2*pi; temp=pi;
while x<=4*pi
    figure; hold on;
    X=temp:pi/10:x;
    for j=1:4
        subplot(2,2,j); plot(X,sin(j*X)./X, 'b:', 'LineWidth', 3);
        switch j
            case 1
                title('Plot of y=sin(x)/x');
            case 2
                title('Plot of y=sin(2x)/x');
            case 3
                title('Plot of y=sin(3x)/x');
            otherwise
                title('Plot of y=sin(4x)/x');
        end
        xlabel('x'); ylabel('y');
    end
    temp=x; x=x+pi;
end
```

Table 11: Nested conditional and loop statements.

4.3.4. Additional Control Flow Statements

Matlab has additional control flow statements that can be used to write an executable script or function. These are more specialized statements that are listed in Table 12. You can learn more details via the online help.
<table>
<thead>
<tr>
<th>Statement</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>break</td>
<td>Terminates the execution of a for or while loop. Statements in the loop that appear after the break statement are not executed.</td>
</tr>
<tr>
<td>continue</td>
<td>Passes control to the next iteration of the for or while loop in which it appears, skipping any remaining statements in the body of the loop.</td>
</tr>
<tr>
<td>try ... catch</td>
<td>Changes flow control if an error is detected during execution.</td>
</tr>
<tr>
<td>return</td>
<td>Causes a normal return to the invoking function or to the keyboard. It also terminates keyboard mode.</td>
</tr>
</tbody>
</table>

**Table 12:** Additional *control flow* statements.
5. m-files: Scripts and Functions

Up to this point, all commands passed to Matlab, were done through the `command window`. But when a segment of programming code is composed from several different commands, then it is time consuming, inefficient and quite difficult to continue using the `command window`. To alleviate this peculiarity, Matlab allows for the creation of functions and scripts by using an `Editor/Debugger` that looks like a text editor.

As said, functions and scripts can be created with the use of the Matlab’s `Editor/Debugger`. You only have to write the command lines, save the file in a directory of your choice and execute it by typing its name to the `command window` and pressing `[Enter]` (you have to make sure that the Matlab’s `current directory` and the directory in which you save the file are the same).

Although m-files represent a text file and it is possible to use any text editor to create a function or a script (as long as the extension of the file is *.m this can work), it is preferable to use the default editor because: i) when you save a file it automatically takes the extension *.m, ii) the reserved words like: for, while, function, end etc are blue colored whilst comments that make the programming code more readable are green colored, and iii) it automatically communicates with the `command window` and allows to run a script via a command found in the `Debug` menu (see the menu bar of `Editor/Debugger`).

5.1 m-files: Functions

Functions are m-files that can accept input arguments and return output arguments. The name of the m-file and of the function should be the same. Functions operate on variables within their own workspace, separate from the workspace you access at the Matlab `command prompt`. Functions are useful for extending the existing Matlab language for personal applications. For example a practitioner might want to write various functions that return...
the theoretical price of a call option contract according to various options pricing models. Options pricing models like the Black and Scholes, the Merton’s Jump’s diffusion, the displaced diffusion model, Heston’s stochastic volatility model, and other can easily be implemented via a different function.

A simple example of a function named as DiagExtract.m that takes as input a square matrix and returns in a vector the elements of its main diagonal is exhibited in Figure 12.

**Figure 12:** An example of a function

For the function DiagExtract note the followings: 1) if after executing your function you get an error, then read the error message that it is returned to the command window (this is very helpful in finding where to look for the
error); you have to return to the Editor to debug the error, re-save the function and try to run it again. If you debug your function or add something, then you have to re-save the file in order to take place the changes (note that if unsaved changes are made in a function file, an asterisk (*) appears next to the function name in the Editor’s title bar (so in Figure 12, the functions has not been saved); ii) although the input argument of the function is a matrix named X and the output is a vector named V, when you call this function from a script or from the command window, you can give any names you like. For example in the command window enter:

“[DiagElements]=DiagExtract(magic(4))”

to see the result; iii) After the function is executed, none of the variables that were used are saved in the workspace; iv) If after each command line you do not place the semi-colon “;” then when the function is executed, you can observe in the command window the result of these command lines (this is useful to be done when you try to debug a function); v) observe on Figure 12 that on the bottom of the Editor/Debugger there is a predefined caption area that indicates that the current m-file is a function with the name DiagExtract.

5.1.1 Basic Parts of a Function
A function is composed from the following parts (most of this section is taken/copied from [2])

5.1.1.1 Function Definition Line
The function definition line informs Matlab that the m-file contains a function, and specifies the argument calling sequence of the function. For example, the function definition line for the average function (that computes the average of a vector) could be:
Matlab function names have the same constraints as variable names. The name must begin with a letter, which may be followed by any combination of letters, digits, and underscores. Making all letters in the name lowercase is recommended as it makes your m-files portable between platforms (of course the use of upper case assures that the functions names you create do not resemble any other Matlab function and/or reserved word). The name of the text file that contains a Matlab function consists of the function name with the extension .m appended. For example, “DiagExtract.m”.

If the filename and the function definition line name are different, the internal (function) name is ignored. Thus, if “DiagExtract.m” is the file that defines a function named: diagonal_extraction, you would invoke the function by typing in the command window:

“DiagExtract”

Concerning function arguments, if the function has multiple output values, enclose the output argument list in square brackets “[ ]”. Input arguments, if present, are enclosed in parentheses “( )”. Use commas to separate multiple inputs or output arguments. Here’s a more complicated example:

“function [Abs, Mean, Std] = StatistiS(X, Y, Z)”

If there is no output, leave the output blank:

“function print_results(x)”

or use empty square brackets:
“function [] = print_results (x)”

The variables that you pass to the function do not need to have the same name as those in the function definition line.

5.1.1.2 The H1 Line

The H1 line, so named because it is the first help text line, is a comment line immediately following the function definition line. Because it consists of comment text, the H1 line begins with a percent sign, "%." For the DiagExtract, the H1 line is:

"% DiagExtract returns the diagonal elements of a square matrix (not single values)."

This is the first line of text that appears when a user types help function_name at the Matlab prompt. Further, the lookfor searches and displays only the H1 line of the functions. Because this line provides important summary information about the m-file, it is important to make it as descriptive as possible.

5.1.1.3 The Help Text

You can create online help for your m-files by entering text on one or more comment lines beginning with the line immediately following the H1 line. The help text for the DiagExtract function is:

"% This function takes as input a square matrix named X and does the followings:
% 1. If by mistake the user does not enter a square matrix, it returns
%    an error message
% 2. If a square matrix is correctly entered, then it returns the
%    elements of its main diagonal in vector V"

When you type help function_name, Matlab displays the comment lines that appear between the function definition line and the first non-comment (executable or blank) line. The help system ignores any comment lines that appear after this help block.
5.1.1.4 The Body Text
The function body contains all the Matlab code that performs computations and assigns values to output arguments. The statements in the function body can consist of function calls, programming constructs like flow control and interactive input/output, calculations, assignments, comments, and blank lines.

For example, the body of the DiagExtract function contains a number of simple programming statements:

```
%    Author: Panayiotis Andreou, Sept. 2003

%    This function uses the error built-in function that displays a message and aborts the function's execution

[m n]=size(X); %Saving the size of the matrix

if isempty(X) %Checking if it is an empty matrix
    error('You have not passed a square matrix');
elseif (m~=n) %Checking if matrix is square
    error('You have enter a non square matrix');
elseif (m==n & m==1) %Checking if it is a single value
    error('You have enter a single value');
else
    Ind=(1:n:n^2)+(0:n-1); %Finding the indeces of the diagonal elements
    V=X(Ind); %Extracting to V the diagonal elements
end
```

5.2 Scripts
On the other hand, scripts can operate on existing data in the workspace, or they can create new data on which to operate. Although scripts do not return output arguments, any variables that they create remain in the workspace, to be used in subsequent computations. In addition, scripts can produce graphical output using functions. Scripts are useful for automating a series of steps that are needed to be performed many times (e.g. to create a
script that calls various options pricing function in order to price a call option and compare the pricing accuracy of each model).

Unlike functions, a script has no a specific structure. It includes a number of commands that are serially executed. As long as the command series has a logical interpretation, the script will result to the desire output. Remember that a script does not take and does not return input and output arguments respectively. The vectors and matrices (variables or scalars) are stored in the Matlab’s workspace. An example of a script follows.

Let’s say that we need to create a script with the name “CompuStat.m” that should store in a vector named Data, the mean, standard deviation, median, covariance, minimum and maximum of the main diagonal of a magic square. Figure 13, shows how a version of such a script might look like for an 8-by-8 magic square.

Figure 13: An example of a script
Try to see what this script does. After executing the `script`, figure out what is the difference between the command `disp` and the `fprintf`. Use the command `who` to view the script’s variables that are saved in the workspace. Notice that a script does not have the same structure as a function. Moreover, observe on Figure 13 that on the bottom of the Editor/Debugger there is a predefined caption area that indicates that the current m-file is a script.

**Important Note:** when you start writing scripts and functions that are stored in a certain directory, you occasional need to see what files are included in this directory. To do so, from the command window you can use the `dir` command (the same one as used in the MS-DOS) to view the contents of Matlab’s current directory or what to view all m-files in the directory. Additionally, you can use `cd directory_name` to set the current directory to the one specified and `cd ..` to move to the directory above the current one. Moreover, `delete file_name` deletes the specified file, and `pwd` prints on the screen the current directory.
6. Cells and Structures

Structures are collections of different kinds of data organized by named fields. Cell arrays are a special class of Matlab arrays whose elements consist of cells that themselves contain Matlab arrays. Both structures and cell arrays provide a hierarchical storage mechanism for dissimilar kinds of data. They differ from each other primarily in the way they organize data. You access data in structures using named fields, while in cell arrays data is accessed through matrix indexing operations [2].

6.1 Cells

A cell array is a Matlab array for which the elements are cells, containers that can hold other Matlab arrays. For example, one cell of a cell array might contain a real matrix, another an array of text strings, and another a vector of complex values [2]. Moreover, you can build cell arrays of any valid size or shape, including multidimensional structure arrays. Figure 14 gives an illustration on how a 2-by-2 cell array might look.

![Figure 14: A 2-by-2 cell array](image-url)
To create a cell array use the command: cell. For example if we want A to be a 3-by-3 cell array, then you should use the following syntax:

“A=cell(3,3)”

to get:

“A =
    []   []   []
    []   []   []
    []   []   []
    “

That is, the A has been created to be a 3-by-3 empty cell array. The cells identification is the same as with matrices. The upper left cell location/position is (1,1), the next cell in the same row is (1,2) and so on (single value indexing is allowed also). You can fill a cell array by assigning data to individual cells one at a time always supporting the correct cell location/index. Matlab has two default ways to store data (scalars, vectors, matrices, strings, cells, etc) to cells:

**Cell indexing**

Enclose the cell subscripts in parentheses using standard array notation. Enclose the cell contents on the right side of the assignment statement in curly braces, "{ }" [2]. For example:

“A(1,1) = {[1 4 3; 0 5 8; 7 2 9]};  A(1,2) = {'Anne Smith'};
A(2,1) = {3+7i};  A(2,2) = {[pi:pi/10:pi]}; A(2,3)=[5];
G=cell(2,1); G(1,1)=[[5 6 9]]; G(2,1)={'Hello World'};
A(3,1) = {G}; A(3,3)=[ones(2,2)]; “

After assigning the cells, in the command window type: “A” to get:
Content indexing

Enclose the cell subscripts in curly braces using standard array notation. Specify the cell contents on the right side of the assignment statement [2]. For example:

```
“A{1,1} = [ 1  4  3; 0  5  8; 7  2  9];   A{1,2} = 'Anne Smith';
A{2,1} = 3+7i;   A{2,2} = -pi:pi/10:pi;   A{2,3}=5;
G=cell(2,1); G{1,1}= [5 6 9]; G{2,1}='Hello World';
A{3,1} = G; A{3,3}=ones(2,2); ”
```

produces the same cell array as before.

If you assign data to a cell that is outside the dimensions of the current array, Matlab automatically expands the array to include the subscripts you specify. It fills any intervening cells with empty matrices. For example, the assignment below turns the 3-by-3 cell array A into a 3-by-4 cell array [2] (note that the same holds for vectors and matrices with the only difference that empty elements are assigned with zero values).

```
“A{3,4}="This expands the cell array';”
```

to get:

```
“A =
[3x3 double]    'Anne Smith'    []
[3.0000+ 7.0000i] [1x21 double]    [ 5]
{2x1 cell }        []   [2x2 double]   [1x27 char]”
```
The only difference between the cell command and the cell and content indexing is that by using cell you pre-allocating the cell's dimensions.

You can use content indexing on the right side of an assignment to access some or all of the data in a single cell. Specify the variable to receive the cell contents on the left side of the assignment. Enclose the cell index expression on the right side of the assignment in curly braces. This indicates that you are assigning cell contents, not the cells themselves. For example, to store to B the 3-by-3 matrix that is located in A(1,1), you should do the following:

```
B=cell{1,1};
```

to get:
```
B =
   1     4     3
   0     5     8
   7     2     9
```

and to save in C the 2nd and 3rd elements of the vector that is saved in the (1,1) cell of G which is saved in the (3,1) cell of A you do the following:

```
C=  A{3,1}{1,1}(2:3)
```

to get:
```
C =
   6     9
```

If you like to save in D the cells (1,2) to (2,3) of A (that is to create a 2-by-2 cell array) you do the following:

```
D=A(1:2,2:3)
```

to get:
```
D =
   'Anne Smith'
   []
```
So, if “{ }” are used after a cell array then Matlab returns the array’s contents but if “( )” are used instead, then it returns cells. To access sub-sets of information stored in a cell use first “{ }” to access the cell and after used “( )” to access the info you want.

Before leaving this section, notice the existence of two very useful cell arrays functions. These are: celldisp that recursively displays the contents of a cell array, and cellplot displays the structure of a cell array as nested coloured boxes. Further info for cell arrays can be obtained from the Matlab’s online help facilities.

6.2 Structures

Structures are Matlab arrays with named data container called fields. The fields of a structure can contain any kind of data. For example, one field might contain a text string representing a name, another might contain a scalar representing a billing amount, a third might hold a matrix of medical test results, and so on. Like standard arrays, structures are inherently array oriented. A single structure is a 1-by-1 structure array, just as the value 5 is a 1-by-1 numeric array. You can build structure arrays with any valid size or shape, including multidimensional structure arrays (see figure below) [2].

(Figure copied from [2])

Structures can be created either by using assignment statements or by using the struct function.
**Building structure arrays using assignment statements**

You can build a simple 1-by-1 structure array by assigning data to individual fields. Matlab automatically builds the structure as you go along. For example, create the 1-by-1 patient structure array shown at the beginning of this section [2]:

```matlab
"patient.name = 'John Doe';
patient.billing = 127.00;
patient.test = [79 75 73; 180 178 177.5; 220 210 205]; "
```

and if you type: “patient” in the command window you get:

```matlab
"patient =
    name: 'John Doe'
    billing: 127
    test: [3x3 double] "
```

that is an array containing a structure with three fields. To expand the structure array, add subscripts after the structure name:

```matlab
"patient(2).name = 'Ann Lane';
patient(2).billing = 28.50;
patient(2).test = [68 70 68; 118 118 119; 172 170 169];"
```

and if you again type: “patient” you will get:

```matlab
"patient =
    1x2 struct array with fields:
    name
    billing
    test "
```

The patient structure array now has size [1 2]. Note that once a structure array contains more than a single element, Matlab does not display
individual field contents when you type the array name. Instead, it shows a summary of the kind of information the structure contains.

You can also use the fieldnames function to obtain this information. fieldnames returns a cell array of strings containing field names (see the online help).

As you expand the structure, Matlab fills in unspecified fields with empty matrices so that:

- All structures in the array have the same number of fields
- All fields have the same field names

For example, entering ‘patient(3).name = 'Alan Johnson’ ” expands the patient array to size [1 3]. Now both “patient(3).billing” and “patient(3).test” contain empty matrices. To see this, type: “patient(3)” to get:

```
ans =
    name: 'Alan Johnson'
    billing: []
    test: []
```

**Building structure arrays using the struct function**

You can pre-allocate an array of structures with the struct function. Its basic form is [2]:

```
str_array = struct (’field1’,val1,’field2’,val2, …)
```

where the arguments are field names and their corresponding values. A field value can be a single value, represented by any Matlab data construct, or a cell array of values. All field values in the argument list must be of the same scale (single value or cell array). To create the structure with the patient info shown before you should have the following syntax (the first command line instructs for the creation of a 1-by-3 structure will all fields set to empty):

```
patient(3) = struct (’name’,[],’billing’,[],’test’,[]);
```
patient(1)=struct('name', 'John Doe', 'billing', 127.00, 'test', ...  
[79 75 73; 180 178 177.5; 220 210 205]);

patient(2)=struct('name', 'Ann Lane', 'billing', 28.50, 'test', ...  
[68 70 68; 118 118 119; 172 170 169]);

patient(3)=struct('name', 'Alan Johnson', 'billing', [], 'test', []);

The view an entire field this can be done with the following example syntax:

"structure_name.field_name"

For example type: “patient.name” in the command window to get:

"patient.name
  ans =
  John Doe
  ans =
  Ann Lane
  ans =
  Alan Johnson "

To view the fields data associated with a certain structure entry, the calling syntax should be of the form:

“structure_name(index_of_entry)"

For example type: “patient(2)” in the command window to get:

“ans =
  name: 'Ann Lane'
  billing: 28.5000
  test: [3x3 double] "

To manipulate sub-data on data stored under a fields name for a specific entry, the calling syntax should be of the form:
“structure_name(index_of_entry).field_name( ) ”

For example type: “patient(2).test(2:end,1:end-1)” in the command window to get:

“ans =

    118   118
    172 170 ”

Useful build-in functions that can make the structures handling much easier include:

• fieldnames to get structure field names
• getfield that can be used to get structures’ fields
• isfield that returns a TRUE if field is in structure array
• rmfield that can be used to delete a structure’s field
• setfield that can be used to assign a structure field

Structures and cell arrays are very useful when you need to write a program that handles various data types. It is a challenge to work with such data types. Before attempting writing a segment of programming code based on these make sure that you have spent a lot of time reading the Matlab’s online help.
7. Entering and Saving Data Files
Matlab can be used to both load a file of data from an external source and to save to your PC data-sets that are located in the workspace or that are created from a function. When you have to deal with small amount of data, then you can enter them by hand either in the command window or in the Editor/Debugger. But when you have to work with a large amount of data retrieved by an external source (e.g. you need to process the historical daily prices for S&P 500 from 1985 to 1999, saved in a file that was downloaded from the Internet), then it is an imperative need to use some build-in functions that can read files of data. Although there are different commands that can load in workspace various types of data files, we review the two most important ones.

7.1. Reading Text ASCII Files: The load command
load is extremely useful command for retrieving from an external source a text file in ASCII form. The general calling syntax of this command is:

   “Data= load (filename);

The filename is better to either have the extension *.txt or *.dat because if it does not have any extension, then the load has a different interpretation (see help load for further details). If the file you want to retrieve is in a different location than the one that the Matlab’s current directory looks, then a complete path name should accompanied this command, for example:

   “Data = load (’c:\Documents\Functions\data.txt’)"

If the load command is entered properly, then the contents of the ASCII file will be stored in a matrix with the dimensionality of the data saved in the ASCII file. In their simplest form, ASCII files contain rows and columns of data separated by either a tab or a space. To be accessible by load the file should not contain any text or missing values and all columns and rows
should have the same number of elements (of course the advance user can handle via certain methodology missing values and text in an ASCII file).

To learn on how ASCII text data look, you can use Microsoft® Excel® that can handle such data. For example, enter the following data (Figure 15) in a new Excel® spreadsheet starting from cell (A1):

![Excel Data](image)

**Figure 15: Excel® Data**

Afterwards, go: *File* > *Save As*... and on the “Save As” window (Figure 16) select a `filename` (e.g. `random_number_data`) for the data to be saved, choose the directory you want to save (e.g. `Matlab Examples`) and moreover, on the field “Save as type:” choose: “Text (Tab delimited)(*.txt)”.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.30023</td>
<td>-1.27768</td>
<td>0.244257</td>
<td>1.276474</td>
<td>1.19836</td>
<td>1.733133</td>
</tr>
<tr>
<td>2</td>
<td>-0.23418</td>
<td>1.095023</td>
<td>-1.0867</td>
<td>-0.6002</td>
<td>-1.69043</td>
<td>-1.84631</td>
</tr>
<tr>
<td>3</td>
<td>-0.77351</td>
<td>-2.11793</td>
<td>-0.56792</td>
<td>-0.40405</td>
<td>0.134535</td>
<td>-0.36549</td>
</tr>
<tr>
<td>4</td>
<td>-0.37024</td>
<td>1.342642</td>
<td>-0.08528</td>
<td>-0.18616</td>
<td>-0.51321</td>
<td>1.972212</td>
</tr>
<tr>
<td>5</td>
<td>2.375655</td>
<td>-0.65491</td>
<td>1.661456</td>
<td>-1.5124</td>
<td>0.538948</td>
<td>0.902191</td>
</tr>
<tr>
<td>6</td>
<td>-0.06452</td>
<td>-0.5236</td>
<td>0.675138</td>
<td>-0.38132</td>
<td>0.767611</td>
<td>-1.44419</td>
</tr>
<tr>
<td>7</td>
<td>-1.52157</td>
<td>-0.36288</td>
<td>-0.03248</td>
<td>0.028117</td>
<td>-0.32272</td>
<td>2.194502</td>
</tr>
</tbody>
</table>
The data will be saved in a file with the name: “random_number_data.txt” that can be loaded in Matlab. If by mistake, you save the file and there is an entry at cell (A8), then when you attempt to load it you will get an error.

7.2 Saving Text ASCII Files: The save command
On the very opposite, when you need to save some data in a text file to be able to access it later either by viewing it with Microsoft® Excel® or by re-loading to Matlab, the user can use the save command. The general calling syntax of the save is of the following form:

“save filename.ext variable_name options”

If a path is not provided with filename, this command saves the elements of the variable_name in a file with the complete name filename.ext (where ext stands for the extension of the filename and for consistency try to always use .txt) in the current directory. If the scalar/vector/matrix (you cannot save cell arrays or structures with this command unless first you extract their data in a matrix or a vector variable) that is located in the workspace under the
name `variable_name` has components like: `NaN` or `Inf`, then these are saved as text and not numbers in the saved `txt` file.

Concerning the `options` in the calling syntax, this can take the following values for ASCII data saves (Table 13):

<table>
<thead>
<tr>
<th><code>-ascii</code></th>
<th><code>-double</code></th>
<th><code>-ascii -tabs</code></th>
<th><code>-double -tabs</code></th>
</tr>
</thead>
<tbody>
<tr>
<td>8-digit ASCII format</td>
<td>16-digit ASCII format</td>
<td>delimits with tabs</td>
<td>16-digit ASCII format, tab delimited</td>
</tr>
</tbody>
</table>

**Table 13:** Options associated with save

The use of “`–ascii`” is common and satisfactory for most programming needs.

### 7.3 Other Reading and saving commands

Matlab can also read and save data in the following data formats (Table 14):

<table>
<thead>
<tr>
<th>File Format</th>
<th>Extension</th>
<th>File Content</th>
<th>Read/save Command</th>
<th>Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Text</td>
<td>mat</td>
<td>Saved MATLAB workspace</td>
<td>load/save</td>
<td>Variables in the file</td>
</tr>
<tr>
<td></td>
<td>csv</td>
<td>Comma-separated numbers</td>
<td>csvread/csvwrite</td>
<td>Double array</td>
</tr>
<tr>
<td></td>
<td>dlm</td>
<td>Delimited text</td>
<td>dlmread/dlmwrite</td>
<td>Double array</td>
</tr>
<tr>
<td></td>
<td>tab</td>
<td>Tab-separated text</td>
<td>dlmread/dlmwrite</td>
<td>Double array</td>
</tr>
<tr>
<td>Scientific Data</td>
<td>cdf</td>
<td>Data in Common Data Format</td>
<td>cdfread</td>
<td>Cell array of CDF records</td>
</tr>
<tr>
<td></td>
<td>fits</td>
<td>Flexible Image Transport System data</td>
<td>fitsread</td>
<td>Primary or extension table data</td>
</tr>
<tr>
<td></td>
<td>hdf</td>
<td>Data in Hierarchical Data Format</td>
<td>hdfread</td>
<td>HDF or HDF-EOS data set</td>
</tr>
<tr>
<td>Spreadsheet</td>
<td>xls</td>
<td>Excel worksheet</td>
<td>xlsread</td>
<td>Double or cell array</td>
</tr>
<tr>
<td></td>
<td>Wh1</td>
<td>Lotus 123 worksheet</td>
<td>wkrread</td>
<td>Double or cell array</td>
</tr>
</tbody>
</table>

**Table 14:** File formats that Matlab can handle.

Use the “help fileformat” to see the file formats that Matlab can handle, or use the help to see the calling syntax of a certain read/save command.
8. Case Studies
In the following sub-section, some very useful examples with *functions* and *scripts* are illustrated. Read, implement and execute the codes that are given. Try to change the code. Experiment by adding your own command lines. Of course, for the needs of the course, you can find all needed *functions*, *scripts* and datasets in the folder named as:

“Matlab Examples”

Depending on the place you are, this folder may be in different locations. Send me an email or ask me in class for the exact location of this folder.

8.1 The Black – Scholes- Merton Options Pricing Formula
The Black Scholes Merton (BSM) model expresses the value of an option as a function of the current value of a stock, \( S \), the option’s strike price, \( X \), the option’s time to maturity, \( T \), the volatility, \( \sigma \), of the stock price returns (this is the standard deviation of the log-relative returns of the stock for the past \( n \) days, where \( n \) is usually set equal to 60), the risk free rate, \( r \), that prevails for a maturity \( T \) and the stock’s dividend yield, \( d \). The analytic formula for the a European call option derived by BSM is:

\[
c^{BSM} = S e^{-dT} N(d_1) - X e^{-rT} N(d_2)
\]

where,

\[
d_1 = \frac{\ln(S/X) + (r-d+s^2/2)T}{s\sqrt{T}},
\]

\[
d_2 = d_1 - s\sqrt{T},
\]

and,
\[ N(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}z^2} \, dz \]

represents the cumulative normal distribution function with mean zero and unity standard deviation (the Matlab built-in function is named as: normcdf).

The value of a European put option can also be defined via the BSM as:

\[ p^{BSM} = Xe^{-rT} N(-d_2) - Se^{-dT} N(-d_1) \]

8.1.1 Implementation of the BSM Formula
We want to create a script named as OptionsBSM.m that calls the user created function BSMPprice to price European call, \( c \), and European put, \( p \), options using the Black, Scholes and Merton model (BSM). The function BSMPprice should take seven (at least six) input parameters (as single values or in the form of a vector) and should return the value (or the vector) of a European call or put. Its calling syntax should look like:

“[Price] = BSMPprice(S, X, T, vol, r, Index, dv)”

“Index” is a string input argument that is set as an additional input argument that takes the values: ‘CALL’ (or any other spelling of the ‘CALL’ such as ‘Call’, ‘CaLL’, etc), or ‘PUT’ (or any other spelling of the ‘PUT’ such as ‘Put’, ‘PuT’, etc) and should be used to define the kind of option type (use the build in function lower to convert the “Index” to lower case). Note the “Index” precedes “dv” because “dv” should be an optional input (will come back to this in a while).

To validate the formula, via the script create a table that tabulates the call and put values against the moneyness ratio, \( S \), and maturity, \( T \). The data should be loaded from a text file (that the user creates either by hand or via
a function) named as “options_data.txt” in which the columns should contain S, X, T, s, r, d. The data values are given below:

\[
S = \{80, 85, 90, 95, 100, 105, 110, 115, 120\},
\]
\[
T = \{0.1, 0.15, 0.20, 0.25\}
\]
\[
s = \{0.1, 0.2, 0.3\}
\]

and,

\[
X = 100, \ r = 0.05, \ d = 0.02
\]

After creating the text file, you should have a 108-by-6 table since for each value of T and s you have nine different possible values for S.

Note that the function:

- Should check if the correct number of input arguments is supplied and display an error massage otherwise (to do so, use the build-in function nargin; see the help facilities for understanding its use).

- Set dividends equal to zero automatically if no dividend input is supplied or if the dividend input argument is set to be an empty array (use the build in function exist).

- In order to prevent an error, the function should check whether the input parameters are all positive or non-negative (elaborate on the BSM formula to see which inputs should be \( \geq 0 \) and which only \( > 0 \); \( d \) and especially \( r \) with negative values can exist but do not have any economical implication).

- If “Index” takes a different value other than those explained before, an error message should be returned and the program execution should break. Additionally, an error should be returned and program execution should stop if “Index” is a string matrix and not a string vector (“Index” should be either 1-by-4 string vector in the case of a ‘CALL’ or a 1-by-3 in the case of a ‘PUT’).
8.1.2 BSM Derivatives

From the same script as above, call a function with the name: BSMderivatives that returns the partial derivatives of the BSM formula (known as the Greek letters) for both the call and the put value. The partial derivatives are:

**delta:** \( \frac{\partial c}{\partial S} = N(d_1)e^{-dT} \), \( \frac{\partial p}{\partial S} = -N(-d_1)e^{-dT} \)

**theta:** \( \frac{\partial c}{\partial T} = -\frac{n(d_1)Ss}{2\sqrt{p}} - rxe^{-rT} N(d_2) \)
\( \frac{\partial p}{\partial T} = -\frac{n(d_1)Ss}{2\sqrt{p}} + rxe^{-rT} N(-d_2) \)

**gamma:** \( \frac{\partial^2 c}{\partial S^2} = \frac{n(d_1)}{Ss\sqrt{T}} \), \( \frac{\partial^2 p}{\partial S^2} = \frac{n(d_1)}{Ss\sqrt{T}} \)

**vega:** \( \frac{\partial c}{\partial s} = S\sqrt{T}n(d_1) \), \( \frac{\partial p}{\partial s} = S\sqrt{T}n(d_1) \)

**rho:** \( \frac{\partial c}{\partial r} = XTe^{-rT}N(d_2) \), \( \frac{\partial p}{\partial r} = -XTe^{-rT}N(-d_2) \)

where,

\[
n(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}
\]

represents the normal probability density function with mean zero and unity standard deviation (the built-in function is named as normpdf). The BSMderivatives calling syntax should be:

“[Derivatives] = BSMderivatives(S, X, T, vol, r, Index, dv)”

The function should make similar checks as before. The output of the function BSMderivatives should be a row vector if the input arguments are single values or a two dimensional array if the input arguments are vectors with the following format:

“Derivatives=[Delta, Theta, Gamma, Vega, Rho]”
8.1.3 BSM Plots and Surfaces

Make the following plots:

- Two subplot figures, one for call options with \( T=0.15 \) and \( s=0.20 \), and one for put options with \( T=0.25 \) and \( s=0.20 \) that plot \( S \) against all partial derivatives. Give titles and axis names.
- A 3D surface of the \( c^{BSM} \) and \( p^{BSM} \) versus the \( S/X \) and \( T \) (note that in order to create a smooth surface you should create a dense mesh-grid) for the ranges:

\[
80 \leq S \leq 120, \ 0.10 \leq T \leq 0.25
\]

for:

\[
X = 100, \ s = 0.20, \ r = 0.05, \ d = 0.02
\]

8.1.4 BSM Implied Volatility

The only unobservable parameter related with the BSM formula when we deal with real world problems is the volatility measure, \( s \) (although as said before it can be estimated via log-relative reruns of the underlying asset). It is accustomed by options traders to observe a market value for a call or put option and given the observed values of \( S, X, ?, r, \) and \( d \), they derive the value of \( s \) that equates the BSM estimate with the market quote. Such \( s \) value is known as implied volatility. For instance, if \( c^{mrk} \) is the market price of a call option and \( S', X', T', r' \) and \( d' \) are the observable parameters related with the call option, then the BSM implied volatility, \( s_{imp} \), is that value of \( s \) that minimizes the absolute error between \( c^{mrk} \) and \( c^{BSM} \). Analytically this problem is:

\[
\min_{s_{imp}} | g(e) | = | c^{mrk} - c^{BSM} (S', X', T', s_{imp}, r', d') |
\]

Alternatively, think this as solving the following equation:

\[
c^{mrk} = c^{BSM} (S, X, T, s_{imp}, r, d)
\]
for $s_{imp}$. It is known that such equation has no analytic solution, so to solve it someone has to implement a numerical root-finding algorithm. The most well known root finding algorithm for the implied volatility problem is the Newton-Raphson method.

The Newton method is very simple and quite fast method. It can be summarized in three steps:

- **Step #1:** give an initial guess for $s_{imp}$ (e.g. $s_{imp}^0 = 0.1$)

- **Step #2:** perform an algorithm iteration, $k$, based on the following formula:

  $$ s_{imp}^k = s_{imp}^{k-1} - \frac{g(e)}{g'(e)} = s_{imp}^{k-1} - \frac{c_{mrk} - c_{BSM}}{\partial c_{BSM}/\partial S} $$

  $$ s_{imp}^k = s_{imp}^{k-1} + \frac{c_{mrk} - c_{BSM}}{\partial c_{BSM}/\partial S} $$

  for $k=1,2,3,\ldots$

- **Step #3:** If $|g(e)| < e$ then stop and return $s_{imp}^k$, otherwise go to Step #2 and perform one more iteration, $k$, of the algorithm (the $e$ is the desire accuracy, usually set to a very small quantity)

Write a function with the following calling syntax:

```
"[ImpliedVol]=BSMimpliedVol(Qmrk, S, X, T, r, Index, dv, maxNumIter, tol)"
```

where “Qmrk” is the market quote of a call or put option. “maxNumIter” represents the number of iterations and “tol” the desire accuracy (it is a stopping criterion related with the absolute difference of market option value and BSM estimate). “Index” is a single string with values “CALL” and “PUT” (or any other combination that delivers the required word meaning) that is used to distinguish between calls and puts as before. Note that all input
arguments precede the optional ones (the optional are “dv”, “maxNumIter” and “tol”).

Use as inputs the initial data from the loaded txt file (without considering s). The function should make similar checking like previous functions with the additional that:

- If not specified, “maxNumIter” should be equal to 100.
- If not specified, “tol” should be 1e-5.
- At the beginning of the function, it should be tested whether with a small value of s (e.g. s=0.001) the \( c^{BSM} \) and \( p^{BSM} \) are greater than \( c^{mrk} \) and \( p^{mrk} \) respectively; if such condition holds (it can be observed in practice as an arbitrage result), an implied volatility value that minimizes \(|g(e)|\) does not exist, so the function should return a “NaN” value.
- To help the algorithm convergence rate and to save computing time, the starting values for the algorithm should be given from the following approximation suggested by Bharadia, Christofides and Salkin [3]:

\[
\text{for call options: } s_{imp}^{0} = \sqrt{\frac{2p}{?} \left( \frac{c^{mrk} - d}{Xe^{-rT} + d} \right)}, \text{ and,}
\]

\[
\text{for put options: } s_{imp}^{0} = \sqrt{\frac{2p}{?} \left( \frac{p^{mrk} + d}{Xe^{-rT} + d} \right)}
\]

with,

\[d = 0.5(S - Xe^{-rT})\]

To use the implied volatility function, replace the volatility column of the options data that were previously loaded from the text file with random values taken from a uniform distribution on the interval [0, 1] (use the built-in function: rand). Afterwards, use the BSMprice to find the theoretical values for calls and puts. In the following, use the BSMpliedVol to verify that the implied volatility is quite close with
the ones that you have initially used (these are the random values) to 
price calls and puts with the BSM. Note that the implied volatility of 
calls and puts for should match for similar input arguments. Plot the 
difference to visualize that always the dollar difference is less than 
“tol” (given that the algorithm iterations were adequate to secure 
convergence).

8.2 Function Minimization and Plots

There is a variety of optimization algorithms that can be used to minimize (or 
to maximize) a univariate or a multivariate function (note that the 
maximization of a function, \( f(x) \), is the same as the minimization of \( -f(x) \)). 
Two well known and widely used algorithms are the gradient descent, that 
minimizes a function by relying solely on a function’s first partial derivatives 
(gradient vector), and the Newton’s descent algorithm that utilizes a 
function’s second order partial derivatives (Hessian matrix).

The gradient descent algorithm can be summarized as follows:

- **Step #1:** give an initial guess for the coordinates of the minimum point 
  \( x^0 \) (e.g. if the function under consideration has two unknowns \( x_1 \) and 
  \( x_2 \), then \( x \) should be a two element row vector: \( x^0 = [x_1^0, x_2^0] \))

- **Step #2:** perform an algorithm iteration, \( k \), based on the following 
  formula:

  \[
  x^{k+1} = x^k - \alpha f'(x^k)
  \]

  for \( k=1,2,3,\ldots \)

  where \( f(x^k) \) is the value of the function at \( k^{th} \) iteration at point \( x \) and 
  \( f'(x^k) \) is a row vector that includes the first order partial derivatives of 
  \( f(.) \) w.r.t \( x \)'s at the \( k^{th} \) iteration. The parameter \( \alpha \) is a positive scale 
  function that lies between 0 and 1 and is a learning rate that sets the 
  step size. Large values of \( \alpha \) force the algorithm to oscillate around the 
  minimum point or even diverge, whereas small values of \( \alpha \) lead to more
stable convergence but with extremely slow rate. Typical values of a range between 0.01 and 0.15 depending on the faced problem.

Specifically, $f'(x)$ for a function with $n$ variables is:

$$f'(x) = \begin{bmatrix} \frac{\partial f(x)}{\partial x_1} & \frac{\partial f(x)}{\partial x_2} & \ldots & \frac{\partial f(x)}{\partial x_n} \end{bmatrix}$$

- **Step #3**: Increase: $k \leftarrow k+1$ and if the current iteration index $k$ is larger than the maximum number of iterations or if $\|f'(x^k)\| < e$ then stop and return $x^k$, otherwise go to Step #2 and perform one more iteration of the algorithm (the $e$ is the desire accuracy, usually set to a small quantity such as $1e^{-6}$, whereas the maximum number of iterations depends solely by the experience of the researcher).

The Newton Descent algorithm can be summarized as follows:

- **Step #1**: give an initial guess for the coordinates of the minimum point $x^0$ (e.g. if the function under consideration has two unknowns $x_1$ and $x_2$, then $x$ should be a two element row vector: $x^0=[x^0_1, x^0_2]$)
- **Step #2**: perform an algorithm iteration, $k$, based on the following formula:

$$x^{k+1} = x^k - f'(x^k)[f''(x^k)]^{-1}$$

for $k=1,2,3,\ldots$

where $f(x^k)$ is the value of the function at $k^\text{th}$ iteration at point $x$.

$f'(x^k)$ is a row vector that includes the first order partial derivatives of $f(.)$ w.r.t $x$’s at the $k^\text{th}$ iteration and $[f''(x^k)]^{-1}$ is a square matrix that represents the inverse of the second order partial derivatives of $f(.)$ w.r.t $x$’s (Hessian matrix) at the $k^\text{th}$ iteration.
Specifically, \( f''(x) \) for a function with \( n \) variables is:

\[
\frac{\partial^2 f(x)}{\partial x_1^2} \quad \frac{\partial^2 f(x)}{\partial x_1 \partial x_2} \quad \ldots \quad \frac{\partial^2 f(x)}{\partial x_1 x_n} \\
\frac{\partial^2 f(x)}{\partial x_2 \partial x_1} \quad \frac{\partial^2 f(x)}{\partial x_2^2} \quad \ldots \quad \frac{\partial^2 f(x)}{\partial x_2 x_n} \\
\vdots \quad \vdots \quad \vdots \quad \ddots \\
\frac{\partial^2 f(x)}{\partial x_n \partial x_1} \quad \frac{\partial^2 f(x)}{\partial x_n \partial x_2} \quad \ldots \quad \frac{\partial^2 f(x)}{\partial x_n^2}
\]

- **Step #3:** Increase: \( k \leftarrow k+1 \) and if the current iteration index \( k \) is larger than the maximum number of iterations or if \( \| f'(x^k) \| \| f''(x^k) \|^{-1} \|< e \) then stop and return \( x^{k+1} \), otherwise go to Step #2 and perform one more iteration of the algorithm (the \( e \) is the desired accuracy, usually set to a small quantity such as \( 1e-6 \), whereas the maximum number of iterations depends solely by the experience of the researcher).

Both the gradient descent and the Newton descent algorithms have inherent problems with their implementation. For example, the user should “magically” define the correct value of the learning parameter \( a \) in order to succeed a robust convergence of the algorithm when it comes to implement the gradient descent algorithm, or concerning the Newton descent one, when the Hessian matrix is singular and its inverse does not exist then it is not applicable. Moreover, there is no way to secure that the initial point to set up the algorithm will eventually help in reaching the minimum point. Most of the times and when the user does not have a detail overview of the faced problem, the initial value of \( x \) might be too far from the minimum point, preventing in this way the algorithm to converge.

There are many suggestions to suppress these problems (i.e [4], [5]). In here, we will implement these two algorithms just for getting the feeling on the way that a minimization algorithm can be used. The user can later improve the functions that will be illustrated according to his/her needs.
To implement the previous algorithms, the user should be in a position to find the gradient vector and the Hessian matrix. Although it is better to work with the exact first and second order partial derivatives of the function via their functional form, a more flexible, practical and quite accurate way is to create two functions in Matlab that calculate these with two sided finite differencing methods. That is, instead of evaluating a function at a point \( x \) by using the functional form of the gradient vector and the Hessian matrix, it is easier to find these information via numerical differentiation (two sided finite differencing).

The centred and evenly spaced finite difference approximation of the first order partial derivatives (gradient vector) of a function \( f \) at point \( x \) is (for simplicity assume a two variable function with \( x_1 \) and \( x_2 \) to be the unknown variable – the extension to the \( n \) dimensional case is trivial):

\[
\frac{\partial f(x)}{\partial x_j} \approx \frac{f(x_1 + h, x_2) - f(x_1 - h, x_2)}{2h}
\]

\[
\frac{\partial f(x)}{\partial x_2} \approx \frac{f(x_1, x_2 + h) - f(x_1, x_2 - h)}{2h}
\]

where \( h \) is given from a rule of thumb (see [4] page 103),

\[
h = \max(|x|, |J|) \varepsilon^{1/4}
\]

with \( \varepsilon \) to represent machine precision (this is the build-in function \( \text{eps} \)).

The centred and evenly spaced finite difference approximation of the second order partial derivatives (Hessian matrix) of a function \( f \) at point \( x \) is (for simplicity assume a two variable function with \( x_1 \) and \( x_2 \) to be the unknown variable – the extension to the \( n \) dimensional case is trivial):

\[
\frac{\partial^2 f(x)}{\partial x_j^2} \approx \frac{f(x_1 + 2h, x_2) + f(x_1 - 2h, x_2) - f(x_1 - 2h, x_2) - f(x_1 + 2h, x_2)}{4h^2}
\]

with \( \varepsilon \) to represent machine precision.
\[ \frac{\partial f^2(x)}{\partial x_1 \partial x_2} = \frac{f(x_1 + h, x_2 + h) + f(x_1 - h, x_2 - h) - f(x_1 - h, x_2 + h) - f(x_1 + h, x_2 - h)}{4h^2} \]

\[ \frac{\partial f^2(x)}{\partial x_1 \partial x_2} = \frac{\partial f^2(x)}{\partial x_2 \partial x_1} \]

\[ \frac{\partial f^2(x)}{\partial x_2^2} \approx \frac{f(x_1 x_2 + 2h) + f(x_1 x_2 - 2h) - f(x_1 x_2 - 2h) - f(x_1 x_2 + 2h)}{4h^2} \]

where \( h \) is given from a rule of thumb (see [4] page 103),

\[ h = \max(|x|, l)^{\frac{2}{3}} \]

Having in mind the above, write a script with the name: \texttt{FunctionMin.m} that will do the following:

- Make the 3D – surface and the contour plots of the function:

\[ f(x_1, x_2) = e^{x}(4x^2 + 2y^2 + 4xy + 2y + 1) \]

in the area \(-2 \leq x, y \leq 1.5\). The function should be saved in an \textit{m-file} with the name: \texttt{funToFile}. Given that you have created the \textit{m-file} with the functional form of the function, to evaluate it at a point use the build in function: \texttt{feval}. The calling syntax is:

\[
\texttt{"feval(@funToFile,x")}
\]

where \( x \) is a two element row vector (or \textit{m-by-2-array}) with “\( x(1) \)” and “\( x(2) \)” to represent \( x_1 \) and \( x_2 \) respectively.

- Do a new figure that creates the contour plot of this function, with the value of the contours to range between 0 and 80 (for example use: “\( V=\{0:0.25:2, 3:1:10, 20:10:80\}\)”).
• Call a function with the name: GradDescent that implements the gradient descent algorithm for minimizing the aforementioned function. The calling syntax of this function should be:

```
[MinPoints, xVals] = GradDescent(fun, x, lr, maxIterNum, tol)
```

fun is the function name “@funToMin”, x is a row vector with the user’s guess (starting value) about the minimum of the function. The input argument “lr” is the learning parameter at that should be set to 0.1 if not specified by the user. maxIterNum” is as before the maximum number of iterations that should be set to 100 if not specified and “tol” is the tolerance concerning the norm of the updating component of the point x(stopping criterion) and should be set to 1e-6 is not specified. The function returns “MinPoints” that if the algorithm has converged is the point where the function is minimized, and “xVals” that is an m-by-2 array with the set of the values of x at each iteration of the algorithm (m is either equal to: maxIterNum” or a smaller number if the algorithm converges).

This function should call another function with the name: NumerDers and with the following calling syntax:

```
GradsVals = NumerDers(fun, x)
```

that takes as input a function and a single coordinate point and returns the gradient vector at that point. Use the two sided finite difference method explained before. For the above, use as initial point: x^0=[0 0].

After you have called the function, plot on the contour figure the algorithm’s trajectory to the minimum found in “xVals”. Call once more the function with x^0=[-1 -2] and add the new trajectory to the contour figure. Use “lr”=0.25 and “maxIterNum”=200. Use gtext to enter a text that indicates the trajectory.
Call a function with the name: Newton that implements the Newton descent algorithm for minimizing the aforementioned function. The calling syntax of this function should be:

```
[MinPoints, xVals] = Newton(fun, x, maxIterNum, tol)
```

“fun” is the function name “@funToMin”, $x$ is a row vector with the user’s guess (starting value) about the minimum of the function. “maxIterNum” is as before the maximum number of iterations that should be set to 25 if not specified and $tol$ is the tolerance concerning the norm of the updating component of the point $x$ (stopping criterion) and should be set to 1e-6 if not specified. The function returns “MinPoints” that given that the algorithm has converged, it is the point where the minimum is located, and “xVals” that is an $m$-by-2 array with the set of the values of $x$ at each iteration of the algorithm ($m$ is either equal to “maxIterNum” or a smaller number if the algorithm converges earlier).

This function should call another function with the name: NumerHessian.m and with the following calling syntax:

```
HessVals = NumerHessian(fun, x)
```

that takes as input a function and a single coordinate point and returns the Hessian at that point. Use the finite two sided difference method explained before. To avoid the calculation of a singular Hessian that cannot be inverted, after saving the Hessian to a square matrix, set equal to zero all its off diagonal elements. This is a very useful rule of thumb that helps the algorithm convergence and robustness. Use $x^0=\begin{bmatrix}0 & 0\end{bmatrix}$ as initial estimate for the minimum.

After you have called the function, create a new contour figure of the function and plot on it the algorithm’s trajectory to the minimum found in “xVals”. Call once more the function with $x_0=\begin{bmatrix}-1 & -2\end{bmatrix}$ and add the new trajectory to the contour figure. Use gtext to enter a text that indicates the trajectory.
8.3 Portfolio Optimization

Many times, a researcher needs to create a portfolio of securities that for a
desire level of return/profit, it bears the least/minimum risk. The portfolio
optimization problem has been defined since Markowitz (1952) who assumed
that the risk associated with a portfolio can be measured with variance.

The researcher objective is do define via an optimization programming
methodology the weights that each of the alternative assets should have in
the portfolio. By construction, the portfolio return is linear w.r.t to the
weights of the assets and it is given from the following relation:

\[ E(r_p) = W^T e \]

where \( r_p \) is the weighted average return of the portfolio and \( E(.) \) is the
expectation operation. The weight representation of each asset in the
portfolio is given by \( W \) and the expected return of each asset is given by \( e \) as
follows:

\[
W = \begin{bmatrix}
w_1 \\
w_2 \\vdots \\
w_n
\end{bmatrix}
\quad \text{and} \quad
e = E\begin{bmatrix}
r_1 \\
r_2 \\vdots \\
r_n
\end{bmatrix}
\]

where \( n \) is the number of available assets. Contrary, the volatility of the
portfolio w.r.t the weights is a high nonlinear function given by the following
relation:

\[ s_p^2 = W^T OW \]

where \( s_p \) represents the weighted average volatility (standard deviation) of
the portfolio, and \( O \) is the assets’ variance-covariance matrix:
$$O = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1} & s_{n2} & \cdots & s_{nn} \end{bmatrix}$$

with $s_{ij} = \text{cov}(r_i, r_j)$. Additionally, for the portfolio optimization problem, it is required that the sum of the assets weights should be added to one:

$$W^TN_I = 1$$

with $N_I$ to be a column vector with unity elements.

From the above it is obvious that the portfolio optimization problem can be solved via the minimization of a quadratic volatility function subjected to linear constrains, one related with the portfolio expected return and another that assures that all available funds are invested. There are several convenient mathematical methods for solving this problem. In here, in first place, it is illustrated the one that is called the method of the Langrangean multiplier that allows the solution of the general portfolio problem with short sales with the requirement that the constraints must be in equality form.

When you take the PBA 521: Financial Theory course, you will have to solve such kind of problems.

The example shown below is copied from the class notes of the above course offered in 2001 by Professor Spiros Martzoukos. It is the case of three risky assets without a risk-free rate (it is trivial to generalize it to $n$ risky assets). The inclusion of the risk free rate is easy; by treating the risk-free rate as a "risky asset" with zero volatility and zero covariance with all the other assets.

The problem formulation that achieves an expected return equal to $R$ follows:

$$\min_{W_I} W^TOW$$

s.t.
\[ W^T e = R \]
\[ W^T N_I = l \]

and for the case of the three assets in an analytic form:

\[
\min_{w_i} w_A^2 s_A^2 + w_B^2 s_B^2 + w_C^2 s_C^2 + 2w_A w_B s_{AB} + 2w_A w_C s_{AC} + 2w_B w_C s_{BC}
\]

s.t.

\[ w_A E(r_A) + w_B E(r_B) + w_C E(r_C) = R \]
\[ w_A + w_B + w_C = l \]

**Note**: If we do not use the first constraint and in the absence of a risk-free rate we will get the minimum variance portfolio. It is actually advisable to first get the minimum variance portfolio before we proceed in a real problem. Then, by changing the level of desire return, we trace points on the efficient frontier.

To solve this problem, we make two simplifications to the problem. First we replace everywhere \( w_C \) with \( 1 - w_A - w_B \). Second, we rearrange the constraint:

\[
w_A E(r_A) + w_B E(r_B) + w_C E(r_C) = R
\]

to

\[
R - w_A E(r_A) + w_B E(r_B) + w_C E(r_C) = 0
\]

and since it equals zero, we multiply it with a constant \( ? \) and we get:

\[
?(R - w_A E(r_A) + w_B E(r_B) + w_C E(r_C)) = 0
\]
Since this equals zero, we add it to the objective function that we want to minimize without affecting the results. Finally, we minimize the quadratic function without any constraints. Thus, we have to minimize a function, \( f \) (so it can also be solved with methods that we have seen before). So, after substitution we have:

\[
f = w_A^2 s_A^2 + w_B^2 s_B^2 + (1 - w_A + w_B)^2 s_C^2 + 2w_A w_B s_{AB} + 2w_A (1 - w_A + w_B) s_{AC} + 2w_B (1 - w_A + w_B) s_{BC} + \{(R - w_A E(r_A) - w_B E(r_B) - (1 - w_A + w_B) E(r_C))\}
\]

We need to solve this problem to find the weights of the three assets in the portfolio. We thus first find the solution to \( w_A, w_B \) and \( \? \). The way to solve the above problem is now easy. We take the partial derivatives each time in respect to one of the three unknowns and set them equal to zero, and we will derive a set of three equations with three unknowns. The first is the partial derivative of \( f \) w.r.t \( w_A \), the second w.r.t \( w_B \) and the third w.r.t the Langrangean multiplier \( \? \).

The first equation gives:

\[
2w_A s_A^2 + (2w_A + 2w_B - 2)s_C^2 + 2w_B s_{AB} + (2 - 4w_A - 2w_B)s_{AC} - 2w_B s_{BC} - \?E(r_A) - \?E(r_C) = 0
\]

The second equation gives:

\[
2w_B s_B^2 + (2w_A + 2w_B - 2)s_C^2 + 2w_A s_{AB} - 2w_A s_{AC} + (2 - 2w_A - 4w_B)s_{BC} - \?E(r_B) + \?E(r_C) = 0
\]

The third equation gives:

\[
R - w_A E(r_A) - w_B E(r_B) - (1 - w_A - w_B) E(r_C) = 0
\]

To work with this case study, assume the following data:
Write a script with the name: MeanVarMin.m that will do the following:

- Saves in a column vector the weights for the solution of the above mean-variance minimization problem given by:

\[
\begin{bmatrix}
0.05 \\
0.10 \\
0.15
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.25 & 0.15 & 0.17 \\
0.15 & 0.21 & 0.09 \\
0.17 & 0.09 & 0.28
\end{bmatrix}
\]

Find its expected return and its standard deviation.

- Create an m-file with the analytic expression of the function \( f \) with the name: portFun.m with the following syntax:

“f=portFun(x,r,V,R)”

with \( x \) to represent a three element vector, \( x = [w_A, w_B, w_C] \), \( r \) to be the vector of the assets expected return, \( V \) the variance covariance matrix and \( R \) the desire portfolio expected return.

- Assuming the inexistence of a risk-free rate, create a plot with the minimum variance opportunity set (current efficient frontier) for values of \( R \) that range between 1% and 40%. The figure should plot the standard deviation against the portfolio expected return. To do so,
change slightly the Newton function so that its calling syntax becomes:

“[MinPoints] = NewtonPort(fun, x, r, V, R)”

with \( r \) to be the column vector of the assets’ expected returns, \( V \) the assets’ variance-covariance matrix and \( R \) the desire expected return. Similar changes in order to pass the additional arguments should be done also to \textit{NumerDers.m} (the new should be named as: \textit{NumerDersPort}) and \textit{NumerHessian.m} (the new should be named as: \textit{NumerHessianPort}). Because the portfolio optimization problem is actually the minimization of a quadratic function, the Newton descent algorithm can find its solution with a single iteration. So, alleviate also the rule of thumb according to which the elements of the Hessian matrix except the ones of the main diagonal are set to zero. Use the origin \((0, 0, 0)\) as an initial point.

- Add a risk free asset with expected return 1\%, find and plot the efficient frontier. Create a new \textit{m-file} with the name: \textit{portFunRiskFree.m} that includes the new expression of \( f \) after the inclusion of risk-free rate \( (\text{hint: find which components are zero-valued and ignore them}) \). Create a third figure that combines the two previous ones. If you are familiar with investment theory, you can view various interesting components (e.g. two fund separation theorem, capital market line, etc).
References


